Published by Institute of Physics Publishing for SISSA/ISAS

Received: *June 25, 2004*
Revised: *September 16, 2004*
Accepted: *September 28, 2004*

MHV vertices and fermionic scattering amplitudes in gauge theory with quarks and gluinos

Jun-Bao Wu

School of Physics, Peking University
Beijing 100871, P. R. China
E-mail: jbwu@pku.edu.cn

Chuan-Jie Zhu*

Institute of Theoretical Physics, Chinese Academy of Sciences
P. O. Box 2735, Beijing 100080, P. R. China
E-mail: zhucj@itp.ac.cn

Abstract: The Cachazo-Svrcek-Witten approach to perturbative gauge theory is extended to gauge theories with quarks and gluinos. All googly amplitudes with quark-antiquark pairs and gluinos are computed and shown to agree with the previously known results. The computations of the non-MHV or non-googly amplitudes are also briefly discussed, in particular the purely fermionic amplitude with 3 quark-antiquark pairs.

Keywords: Nonperturbative Effects, Strong Coupling Expansion, QCD.

*Supported in part by fund from the National Natural Science Foundation of China with grant Number 90103004.

JHEP09(2004)063

Contents

1. Introduction

Recently Witten [1] found a deep connection between the perturbative gauge theory and string theory in twistor space [2]. Based on this work, Cachazo, Svrcek and Witten (CSW for short) reformulated the perturbative calculation of the scattering amplitudes in Yang-Mills theory by using the off shell MHV vertices [3]. The MHV vertices they used are the usual tree level MHV scattering amplitudes in gauge theory [4, 5], continued off shell in a particular fashion as given in [3]. (For references on perturbative calculations, see for example [6]–[10]. The 2 dimensional origin of the MHV amplitudes in gauge theory was first given in [11].) Some sample calculations were done in [3], sometimes with the help of symbolic manipulation. The correctness of the rules was partially verified by reproducing the known results for small number of gluons [6].

In two previous works [12, 13] (for recent works, see [14]–[30]), by following the new approach of [3], we have computed the generic gluon googly amplitudes and discussed briefly how the CSW approach can be extended to theories with fermions (see also [22]). In this paper we will present the full set of the CSW rules for gauge theories with quarks and antiquarks (fermions in the fundamental representation) and gluinos (in supersymmetric theory or fermions in the adjoint representation). Although some of the fermionic amplitudes at tree level can be obtained by supersymmetric Ward identities [33, 34, 6, 35] we think it is still worthy to compute these amplitudes directly. Although the CSW rule can be partially understood from the twistor string theory [23], a full understanding of the CSW approach from the conventional field theory is not reached [28].

By using these CSW rules with fermions, we will compute the googly amplitudes with one and two quark-antiquark pairs. We will also compute the googly amplitudes with 2

JHEP09(2004)063

gluinos. For all these amplitudes, we got results which are in agreement with the previously known results. We note that these googly amplitudes are simply the "complex conjugate" of the corresponding MHV amplitudes.

The amplitudes with more than two quark-antiquark pairs are no longer MHV or googly. Nevertheless these amplitudes can be simply computed by using the extended CSW rules. As an example we will analyze the purely fermionic amplitudes with 3 quark-antiquark pairs. There are only 2 different kinds of diagrams and the amplitude can be written down very simply. We conjecture that this will be the right result for the amplitude.

This paper is organized as follows. In section 2, we review the CSW rule for gauge theory without fermions. In section 3, we give the extended CSW rules for gauge theories with quarks and gluinos. Here we list all the MHV vertices with fermions. Some general relations between the number of vertices and the number of external gluons are also given. These relations are particularly useful for analyzing diagrams with fewer number of external particles with positive helicity. We will use them to draw the possible diagrams for the purely fermionic amplitudes with 3 quark-antiquark pairs. In section 4, we compute the googly amplitude with one quark-antiquark pair. There is only 1 external gluon with positive helicity and there can be an arbitrary number of gluons with negative helicity. In section 5, we compute the googly amplitude with 2 quark-antiquark pairs. In section 6, we compute the googly amplitude with 2 gluinos. The computation for the 4 gluino googly amplitude can also be done and the algebra is more or less the same as before. A technical and lengthy proof in section 4 is relegated to appendix A.

2. Review of the CSW approach to perturbative gauge theory

First let us review the rules for calculating tree level gauge theory amplitudes as proposed in [3]. Here we follow the presentation given in [12, 13] closely. We will use the convention that all momenta are outgoing. By MHV (with gluons only), we always mean an amplitude with precisely two gluons of negative helicity. If the two gluons of negative helicity are labelled as r, s (which may be any integers from 1 to n), the MHV vertices (or amplitudes) are given as follows:

$$V_n = \frac{\langle \lambda_r, \lambda_s \rangle^4}{\prod_{i=1}^n \langle \lambda_i, \lambda_{i+1} \rangle} . \tag{2.1}$$

For an on shell (massless) gluon, the momentum in bispinor basis is given as:

$$p_{a\dot{a}} = \sigma^\mu_{a\dot{a}} p_\mu = \lambda_a \tilde{\lambda}_{\dot{a}} . \tag{2.2}$$

For an off shell momentum, we can no longer define λ_a as above. The off-shell continuation given in [3] is to choose an arbitrary spinor $\tilde{\eta}^{\dot{a}}$ and then to define λ_a as follows:

$$\lambda_a = p_{a\dot{a}} \tilde{\eta}^{\dot{a}} . \tag{2.3}$$

For an on shell momentum p, we will use the notation λ_{pa} which is proportional to λ_a:

$$\lambda_{pa} \equiv p_{a\dot{a}} \tilde{\eta}^{\dot{a}} = \lambda_a \tilde{\lambda}_{\dot{a}} \tilde{\eta}^{\dot{a}} \equiv \lambda_a \phi_p . \tag{2.4}$$

JHEP09(2004)063

As demonstrated in [3], it is consistent to use the same $\tilde{\eta}$ for all the off shell lines (or momenta). The final result is independent of $\tilde{\eta}$.

By using only MHV vertices, one can build a tree diagram by connecting MHV vertices with propagators. For the propagator of momentum p, we assign a factor $1/p^2$. Any possible diagram (involving only MHV vertices) will contribute to the amplitude. As proved in [3], a tree level amplitude with n_- external gluons of negative helicity must be obtained from an MHV tree diagram with $n_- - 1$ vertices. Another relation was given in [12],

$$n_+ = \sum_i (i-3)\, n_i + 1\,, \tag{2.5}$$

when n_+ is the number of the external gluons with positive helicity, and n_i is the number of the vertices with exactly i line. The other relation stated in the above is:

$$n_- = \sum_i n_i + 1\,. \tag{2.6}$$

From eq. (2.5) we can see that a tree level amplitude with n_+ external gluons of positive helicity will have no contribution from any diagram containing an MHV vertex with more than $n_+ + 2$ lines (not necessarily all internal). For the googly amplitude we have $n_+ = 2$. Any contributing diagram will have exactly one MHV vertex with 4 lines. The rest MHV vertices are all with 3 lines.

For future use, let us recall 2 formulas presented in [12] about the off shell amplitude with $n_+ = 1$. When the first particle with momentum p_1 is off shell and has positive helicity, the amplitude is

$$V_n(1+, 2-, \ldots, n-) = \frac{p_1^2}{\phi_2 \phi_n} \frac{1}{[2,3][3,4]\cdots[n-1,n]}\,. \tag{2.7}$$

When it has negative helicity, the formula is

$$V_n(1-, 2-, \ldots, r+, \ldots, n-) = \frac{\phi_r^4 p_1^2}{\phi_2 \phi_n} \frac{1}{[2,3][3,4]\cdots[n-1,n]}\,. \tag{2.8}$$

We stress the fact that the above off shell amplitudes are proportional to p_1^2 and they vanish when p_1 is also on shell ($p_1^2 = 0$).

3. The fermionic MHV vertices

For gauge theory coupled with quark and antiquark, we can decompose an amplitude into partial amplitudes with definite color factors [6]. For simplicity we will assume that all quarks have different flavors. We will indicate what should be changed if there are identical quark-antiquark pairs. Also we will assume the gauge group to be U(N) instead of SU(N). For a connected diagram with m pair of quark-antiquark, the color factor is

$$(T^{a_1}\cdots T^{a_{n_1}})_{i_1\bar{i}_2}(T^{a_{n_1+1}}\cdots T^{a_{n_2}})_{i_2\bar{i}_3}\cdots(T^{a_{n_{m-1}+1}}\cdots T^{a_n})_{i_m\bar{i}_1}\,, \tag{3.1}$$

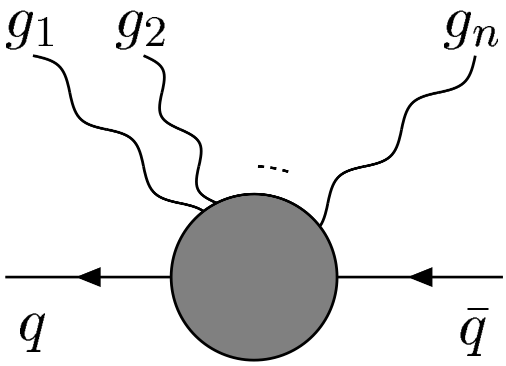

Figure 1: The graphic representation for the single pair of quark-antiquark partial amplitude. Gluons are emitted from one side of the fermion line only.

for a particular ordering of the quark-antiquarks and gluons [36]. The corresponding partial amplitude is denoted as:

$$A(\Lambda^{h_1}_{q_1}, g_1, \ldots, g_{n_1}, \Lambda^{-h_2}_{\bar{q}_2}, \Lambda^{h_2}_{q_2}, g_{n_1+1}, \ldots, g_{n_2}, \ldots, \Lambda^{-h_m}_{\bar{q}_m}, \Lambda^{h_m}_{q_m}, g_{n_{m-1}+1}, \ldots, g_n, \Lambda^{-h_1}_{\bar{q}_1})\,. \quad (3.2)$$

For amplitudes with connected Feynman diagrams, the quark-antiquark color indices $(i, \bar{i})$ must form a ring of length exactly m. There is no disconnected rings of shorter length, like in the color factor $(T^{\cdots})_{i_1\bar{i}_2}(T^{\cdots})_{i_2\bar{i}_1}\ (T^{\cdots})_{i_3\bar{i}_4}(T^{\cdots})_{i_4\bar{i}_3}$. This can be proved by induction with the number of pairs m.

The complete amplitude is obtained first by summing over all possible partitions of n gluons into m parts and their permutations. Then there is another summation over all possible permutations between quark-antiquark pairs. If there are identical quark-antiquark pairs, we should do one more summation over all possible permutation between identical antiquarks with a minus sign if the permutation is odd. Of course, the summations of different flavor identical quark-antiquark pairs should be done separately.

For a single quark-antiquark pair the color factor is $(T^{a_1}\cdots T^{a_n})_{i\bar{i}}$. The partial amplitude is denoted as $A_{n+2}(\Lambda^h_q, g_1, \ldots, g_n, \Lambda^{-h}_{\bar{q}})$.[1] It is represented as in figure 1. We note that gluon lines are emitted only from one side of the (connected) quark-antiquark line. We will stick to this rule also for multi-pair of quark-antiquark diagrams.

There are 2 MHV vertices with quark-antiquarks, one for a single pair of quark-antiquark and one for two quark-antiquark pairs which are shown in figure 2. There is no MHV vertex for 3 or more pair of quark-antiquark. All these (non-MHV) amplitudes should be computed from the above MHV vertices by drawing all possible (connected) diagrams with only MHV vertices.

The explicit formulas for these MHV amplitudes are given as follows:

$$A(\Lambda^+_q, g^+_1, \ldots, g^-_I, \ldots, g^+_n, \Lambda^-_{\bar{q}}) = -\frac{\langle q, I\rangle\langle \bar{q}, I\rangle^3}{\langle q, 1\rangle\langle 1, 2\rangle\cdots\langle n, \bar{q}\rangle\langle \bar{q}, q\rangle}\,, \quad (3.3)$$

$$A(\Lambda^-_q, g^+_1, \ldots, g^-_I, \ldots, g^+_n, \Lambda^+_{\bar{q}}) = \frac{\langle q, I\rangle^3\langle \bar{q}, I\rangle}{\langle q, 1\rangle\langle 1, 2\rangle\cdots\langle n, \bar{q}\rangle\langle \bar{q}, q\rangle}\,, \quad (3.4)$$

[1] h denotes the helicity of the quark q. The helicity of the antiquark $\bar{q}$ is $-h$ by helicity conservation along the quark line.

JHEP09(2004)063

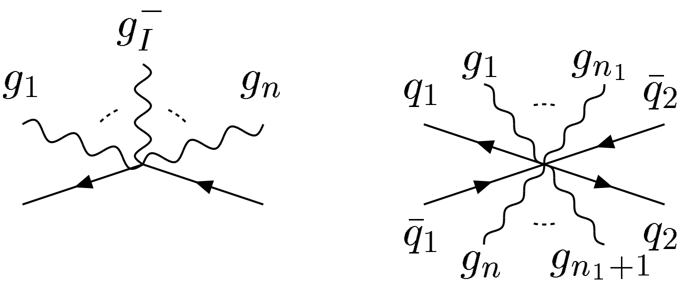

Figure 2: The 2 MHV vertices with quark-antiquarks.

for the single pair of quark-antiquark, and for 2 quark-antiquark pairs:

$$\begin{aligned} & V(\Lambda_{q_1}^{h_1}, g_1^+, \ldots, g_{n_1}^+, \Lambda_{\bar{q}_2}^{-h_2}, \Lambda_{q_2}^{h_2}, g_{n_1+1}^+, \ldots, g_n^+, \Lambda_{\bar{q}_1}^{-h_1}) = \\ & \quad = A_0(h_1, h_2) \frac{\langle q_1, \bar{q}_2 \rangle}{\langle q_1, 1 \rangle \langle 1, 2 \rangle \cdots \langle n_1, \bar{q}_2 \rangle} \times \frac{\langle q_2, \bar{q}_1 \rangle}{\langle q_2, n_1+1 \rangle \cdots \langle n, \bar{q}_1 \rangle}, \end{aligned} \tag{3.5}$$

where $A_0(h_1, h_2)$ is given as follows:

$$A_0(+,+) = \frac{\langle \bar{q}_1, \bar{q}_2 \rangle^2}{\langle q_1, \bar{q}_1 \rangle \langle q_2, \bar{q}_2 \rangle}, \qquad A_0(+,-) = -\frac{\langle \bar{q}_1, q_2 \rangle^2}{\langle q_1, \bar{q}_1 \rangle \langle q_2, \bar{q}_2 \rangle}, \tag{3.6}$$

$$A_0(-,+) = -\frac{\langle q_1, \bar{q}_2 \rangle^2}{\langle q_1, \bar{q}_1 \rangle \langle q_2, \bar{q}_2 \rangle}, \qquad A_0(-,-) = \frac{\langle q_1, q_2 \rangle^2}{\langle q_1, \bar{q}_1 \rangle \langle q_2, \bar{q}_2 \rangle}. \tag{3.7}$$

All these MHV amplitudes are given in terms of the "holomorphic" spinors of the external (on-shell) momenta. So we can use the same off shell continuation given in [3] which we recalled in section 2. By including these fermionic MHV vertices, we can extend the CSW rule of perturbative gauge theory to gauge theories with quarks and antiquarks. The propagator for both gluon and fermion (quark or antiquark) internal lines is just $1/p^2$, as explained in [22]. The only peculiarity with fermions is that helicity is conserved along a fermion line. Because we assume that all quarks have different flavor, the flowing of arrows must follow the directions given exactly in figure 2.

Let us assume that in an MHV diagram, there are n_i purely gluonic MHV vertices with i-lines, n_i^{2f} single pair of quark-antiquark MHV vertices with $(i+2)$-lines (counting also the 2 fermion lines, so actually only i gluon lines), and n_i^{4f} 2 pairs of quark-antiquark MHV vertices with $(i+4)$-lines (counting also the 4 fermion lines, so actually only i gluon lines). For a connected diagram with m quark-antiquark pairs, the number of positive helicity gluon n_+ and the number of negative helicity gluon n_- are given as follows:

$$n_- = \sum_{i \geq 3} n_i + \sum_{i \geq 1} n_i^{2f} + \sum_{i \geq 0} n_i^{4f} - (m-1), \tag{3.8}$$

$$n_+ = \sum_{i \geq 3} (i-3)\, n_i + \sum_{i \geq 1} (i-1)\, n_i^{2f} + \sum_{i \geq 0} (i+1)\, n_i^{4f} - (m-1). \tag{3.9}$$

For googly amplitude with $m = 1$ and $n_+ = 1$, we have

$$n_4 + n_2^{2f} + n_0^{4f} = 1, \tag{3.10}$$

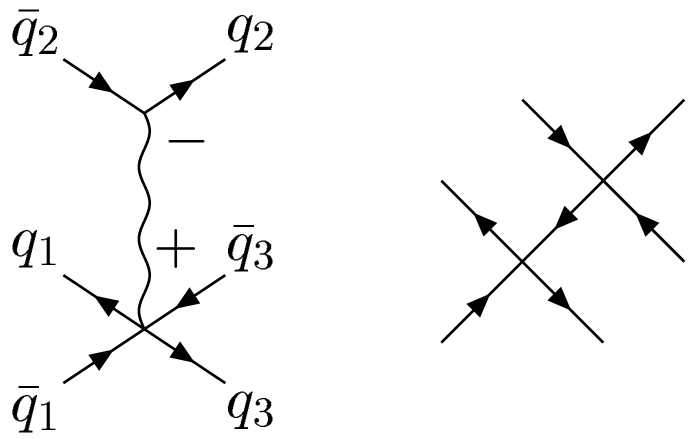

Figure 3: The 2 different kinds of diagrams contributing to the purely fermionic amplitude with 3 quark-antiquark pairs.

and $n_{i>4} = n^{2f}_{i>2} = n^{4f}_{i>0} = 0$. So there is either a single 4 line gluon MHV vertex, or a 4 line single pair of quark-antiquark MHV vertex, or a 4 line double pair of quark-antiquark MHV vertex (which is not possible because we have only 2 external fermion lines). See figures 8 and 9. We will use this result in section 4.

Eqs. (3.8) and (3.9) are quite powerful for analyzing the possible diagrams. These relations are particularly useful for analyzing diagrams with fewer number of external particles with positive helicity. For the purely fermionic amplitudes with 3 quark-antiquark pairs, we found that there are only 2 different kinds of diagrams as shown in figure 3. By using the extended CSW rules, the partial amplitude can be written down very simply as follows:

$$A_6^{CSW}(\Lambda^-_{q_1}, \Lambda^+_{\bar{q}_2}, \Lambda^-_{q_2}, \Lambda^+_{\bar{q}_3}, \Lambda^-_{q_3}, \Lambda^+_{\bar{q}_1}) = \sum_{i=1}^{3} A^i + \sum_{i=1}^{3} \tilde{A}^i \,, \tag{3.11}$$

where

$$\begin{aligned} A^i &= -\frac{\langle q_i, (\bar{q}_i\, q_i)\rangle^3 \,\langle \bar{q}_i, (\bar{q}_i\, q_i)\rangle}{\langle \bar{q}_i, q_i\rangle \,\langle q_i, (\bar{q}_i\, q_i)\rangle \,\langle (\bar{q}_i\, q_i), \bar{q}_i\rangle} \,\frac{1}{(p_{\bar{q}_i} + p_{q_i})^2} \times \\ &\quad \times \frac{\langle q_{i+1}, q_{i+2}\rangle^2}{\langle \bar{q}_{i+1}, q_{i+1}\rangle \,\langle \bar{q}_{i+2}, q_{i+2}\rangle} \,\frac{\langle q_{i+2}, \bar{q}_{i+1}\rangle}{\langle q_{i+2}, (\bar{q}_i\, q_i)\rangle \langle (\bar{q}_i\, q_i), \bar{q}_{i+1}\rangle} \\ &= \frac{\langle q_i, (\bar{q}_i\, q_i)\rangle^2}{\langle \bar{q}_i, q_i\rangle} \,\frac{1}{(p_{\bar{q}_i} + p_{q_i})^2} \,\frac{\langle q_{i+1}, q_{i+2}\rangle^2}{\langle \bar{q}_{i+1}, q_{i+1}\rangle \,\langle \bar{q}_{i+2}, q_{i+2}\rangle} \times \\ &\quad \times \frac{\langle q_{i+2}, \bar{q}_{i+1}\rangle}{\langle q_{i+2}, (\bar{q}_i\, q_i)\rangle \langle (\bar{q}_i\, q_i), \bar{q}_{i+1}\rangle} \,, \end{aligned} \tag{3.12}$$

and

$$\begin{aligned} \tilde{A}^i &= -\frac{\langle q_i, (\bar{q}_i\, \bar{q}_{i+1})\rangle^2}{\langle \bar{q}_i, q_i\rangle \,\langle \bar{q}_{i+1}, (\bar{q}_i\, q_i)\rangle} \,\frac{1}{(p_{\bar{q}_i} + p_{q_i} + p_{\bar{q}_{i+1}})^2} \times \\ &\quad \times \frac{\langle q_{i+1}, q_{i+2}\rangle^2}{\langle (\bar{q}_{i+2}\, q_{i+2}), q_{i+1}\rangle \,\langle \bar{q}_{i+2}, q_{i+2}\rangle} \,, \end{aligned} \tag{3.13}$$

for one set of quark helicities. Here the expression $\langle (ij), k\rangle$ is defined as $\langle \lambda_{p_i} + \lambda_{p_j}, \lambda_k\rangle$, and the indices are understood mod 3. We mention that the six-quark amplitude had been computed before in [31]. For other quark helicities and the proof that the above amplitudes indeed agree with the standard field theory results by using the Feynman rules, we refer to [32].

For supersymmetric theories, there are also gluinos, the super partners of the gluons. Because the gluinos are in the same adjoint representation as the gluons, the color decomposition for amplitudes with both gluons and gluinos would be the same as for purely gluon amplitudes. Compared with gauge theory with quark-antiquarks, there is no difference between gluino and its antiparticle. So we should include also diagrams with gluon lines emitted from both sides of the gluino lines.

The gluino MHV amplitudes can be obtained by using the recursive relation in [5] or by using the supersymmetric Ward identity [33, 34, 6] and are recalled here. The MHV vertex with two gluinos ($r < s$) is:

$$V_n(g_1, \ldots, g_t^-, \ldots, \Lambda_r^-, \ldots, \Lambda_s^+, \ldots, g_n) = \frac{\langle t, r\rangle^3 \langle t, s\rangle}{\prod_{i=1}^n \langle i, i+1\rangle}\,, \tag{3.14}$$

where all gluons except one (denoted as g_t) have positive helicity and the gluinos are denoted as $\Lambda_{r,s}$ with their helicities. As we noted earlier, the helicities along a fermion line must be conserved. The other case when the positive helicity gluino is in front of the negative helicity gluino can be obtained from the above formula by cyclic permutation. One should only note that there is an extra $-$ sign when we change the order of two fermions because of Fermi statistics. So we have:

$$V_n(g_1^+, \ldots, g_t^-, \ldots, \Lambda_s^+, \ldots, \Lambda_r^-, \ldots, g_n^+) = -\frac{\langle t, r\rangle^3 \langle t, s\rangle}{\prod_{i=1}^n \langle i, i+1\rangle}\,, \tag{3.15}$$

for $s < r$ [22]. From the above two equations, we see that the position of the negative helicity gluon is immaterial.

The MHV vertices with 4 gluinos is:

$$V_n(g_1, \ldots, \Lambda_p^-, \ldots, \Lambda_q^-, \ldots, \Lambda_r^+, \ldots, \Lambda_s^+, \ldots, g_n) = -\frac{\langle r, s\rangle^3 \langle p, q\rangle}{\prod_{i=1}^n \langle i, i+1\rangle}\,, \tag{3.16}$$

where all gluons have positive helicity. We note that the above formula is antisymmetric when we exchange two gluinos with the same helicity.

We propose that one can use the above MHV vertices to calculate all the tree-level (partial) amplitudes by extending the CSW approach to gauge theory with quarks and gluinos. We use the same off shell continuation given in [3]. In the following three sections we will test these rules by calculating all the googly amplitudes with quarks and gluinos. As one can see from the intermediate steps of our computations, we obtained quite explicit formulas for some off shell amplitudes. It would be interesting to explore the connection of these amplitudes with the non-MHV vertices introduced in [28].

4. The googly amplitudes with one quark-anti-quark pair

In this section we will compute the googly amplitude with one quark-antiquark pair which we reported briefly in [13]. We will present here the full details of the computations. The other cases with 2 quark-antiquark pairs and the supersymmetric case with gluinos, will be discussed in the next two sections. In this section we will prove the following formulas for the googly amplitudes (only the I-th gluon has positive helicity):

$$A(\Lambda_q^+, g_1^-, \ldots, g_I^+, \ldots, g_n^-, \Lambda_{\bar{q}}^-) = \frac{[q,I]^3[\bar{q},I]}{[q,1][1,2]\cdots[n,\bar{q}][\bar{q},q]}, \tag{4.1}$$

$$A(\Lambda_q^-, g_1^-, \ldots, g_I^+, \ldots, g_n^-, \Lambda_{\bar{q}}^+) = -\frac{[q,I][\bar{q},I]^3}{[q,1][1,2]\cdots[n,\bar{q}][\bar{q},q]}. \tag{4.2}$$

In order to compute the above googly amplitudes, we first calculate the amplitudes with one quark-antiquark pair when all gluons have negative helicity and only one particle is off-shell. This amplitude would vanish when all particles are on shell. So we expect that it is proportional to p^2 (p is the momentum of the only off-shell particle). As one can see from eq. (3.9), for $n_+ = 0$ and $m = 1$, only n_3 and n_1^{2f} could be non-vanishing. So all the contributing diagrams are composed of 3-line MHV vertices only.

To start, let us begin with $n_- = 1$. When all the 3 particles are off shell, the MHV amplitude with one quark-antiquark pair can be written as:

$$A(\Lambda_q^+, g_1^-, \Lambda_{\bar{q}}^-) = \frac{\langle q,1\rangle^2}{\langle \bar{q},q\rangle} = \langle q,1\rangle = \langle 1,\bar{q}\rangle = \langle \bar{q},q\rangle, \tag{4.3}$$

$$A(\Lambda_q^-, g_1^-, \Lambda_{\bar{q}}^+) = -\frac{\langle 1,\bar{q}\rangle^2}{\langle \bar{q},q\rangle} = -\langle q,1\rangle = -\langle 1,\bar{q}\rangle = -\langle \bar{q},q\rangle. \tag{4.4}$$

The amplitude $A(\Lambda_q^+, g_1^-, \ldots, g_n^-, \Lambda_{\bar{q}}^-)$ when Λ_q is off shell and has positive helicity, is given as follows:

$$A(\Lambda_q^+, g_1^-, \ldots, g_n^-, \Lambda_{\bar{q}}^-) = \frac{p_q^2}{\phi_1}\frac{1}{[1,2][2,3]\ldots[n-1,n][n,\bar{q}]}. \tag{4.5}$$

As in [12], we will prove this formula by mathematical induction with the number of gluons $n = n_-$.

When $n = 1$, from momentum conservation, we have

$$\lambda_q + \lambda_1\phi_1 + \lambda_{\bar{q}}\phi_{\bar{q}} = 0, \tag{4.6}$$

and so

$$A(\Lambda_q^+, g_1^-, \Lambda_{\bar{q}}^-) = \frac{\langle 1,\bar{q}\rangle}{\phi_1} = \frac{p_q^2}{\phi_1}\frac{1}{[1,\bar{q}]}. \tag{4.7}$$

This shows that eq. (4.5) is true for $n = 1$. Now we assume that it is valid for all $k < n$. We will show that it is also valid for $k = n$.

JHEP09(2004)063

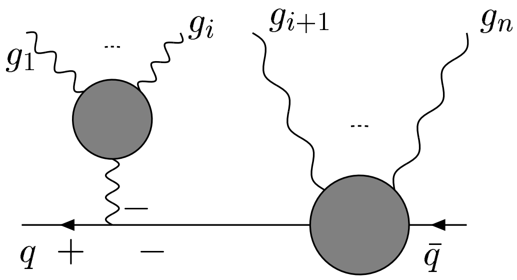

Figure 4: The decomposition for the off shell amplitude with one quark-antiquark pair when the gluons are all with positive helicities and the quark is off-shell.

In order to compute the amplitude with n gluons we use the diagram decomposition as shown in figure 4. By using the assumed result for less number of gluons and also eq. (2.7), we have

$$A(\Lambda_q^+, g_1^-, \ldots, g_n^-, \Lambda_{\bar{q}}^-) = \sum_{i=1}^{n} \frac{p^2}{\phi_1\phi_i} \frac{1}{[1,2]\cdots[i-1,i]} \times \frac{1}{p^2} \times \frac{\langle\lambda_p, \lambda_k\rangle^2}{\langle\lambda_k, \lambda_q\rangle} \times \\ \times \frac{1}{k^2} \times \frac{k^2}{\phi_{i+1}} \frac{1}{[i+1,i+2]\cdots[n-1,n][n,\bar{q}]}, \tag{4.8}$$

where

$$p = \sum_{l=1}^{i} p_l\,, \qquad \lambda_p = \sum_{l=1}^{i} \lambda_l \phi_l\,, \tag{4.9}$$

$$k = \sum_{l=i+1}^{n+1} p_l\,, \qquad \lambda_k = \sum_{l=i+1}^{n+1} \lambda_l \phi_l\,, \tag{4.10}$$

and the index $n+1$ (whenever it appears) refers to the antiquark $\Lambda_{\bar{q}}$. We note that the degenerate cases for $i=1$ and $i=n$ are also included correctly in the above sum over i.

By using eq. (4.3), we have

$$A(\Lambda_q^+, g_1^-, \ldots, g_n^-, \Lambda_{\bar{q}}^-) = \frac{1}{\phi_1} \frac{1}{[1,2][2,3]\cdots[n-1,n][n,\bar{q}]} \sum_{i=1}^{n} \frac{[i,i+1]}{\phi_i\phi_{i+1}} \langle\lambda_p, \lambda_k\rangle\,. \tag{4.11}$$

By using the following identity,

$$\sum_{i=2}^{n-1} \frac{[i,i+1]}{\phi_i\phi_{i+1}} \langle\lambda_{p_2+\cdots+p_i}, \lambda_{p_{i+1}+\cdots+p_n}\rangle = (p_2+p_3+\cdots+p_n)^2\,, \tag{4.12}$$

given in [12], we have

$$\sum_{i=1}^{n} \frac{[i,i+1]}{\phi_i\phi_{i+1}} \langle\lambda_p, \lambda_k\rangle = (p_1+\cdots+p_n+p_{\bar{q}})^2 = p_q^2\,. \tag{4.13}$$

By using this result in eq. (4.11), we have

$$A(\Lambda_q^+, g_1^-, \cdots, g_n^-, \Lambda_{\bar{q}}^-) = \frac{p_q^2}{\phi_1} \frac{1}{[1,2][2,3]\cdots[n-1,n][n,\bar{q}]}\,, \tag{4.14}$$

which is the result of eq. (4.5) for n gluons. This completes the proof of eq. (4.5).

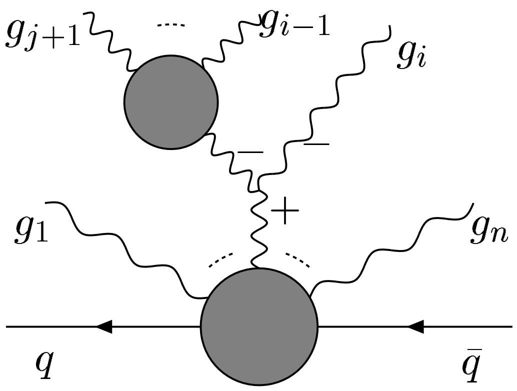

Figure 5: This diagram is characterized with g_i attaching to a 3-gluon MHV vertex. g_{i-1}'s are disconnected from the fermion line by this vertex.

By using the same method, one can obtain the following result:

$$A(\Lambda_q^-, g_1^-, \ldots, g_n^-, \Lambda_{\bar{q}}^+) = -\frac{p_q^2 \phi_{\bar{q}}^2}{\phi_1} \frac{1}{[1,2][2,3]\cdots[n-1,n][n,\bar{q}]}\,, \tag{4.15}$$

when the quark Λ_q is off shell and has negative helicity.

When the anti-quark $\Lambda_{\bar{q}}$ is off shell, one can similarly obtain the following results:

$$A(\Lambda_q^+, g_1^-, \ldots, g_n^-, \Lambda_{\bar{q}}^-) = \frac{p_{\bar{q}}^2 \phi_q^2}{\phi_n} \frac{1}{[q,1][1,2][2,3]\cdots[n-1,n]}\,, \tag{4.16}$$

$$A(\Lambda_q^-, g_1^-, \ldots, g_n^-, \Lambda_{\bar{q}}^+) = -\frac{p_{\bar{q}}^2}{\phi_n} \frac{1}{[q,1][1,2][2,3]\cdots[n-1,n]}\,. \tag{4.17}$$

When the off shell particle is one of the gluons g_i, the results are:

$$A(\Lambda_q^+, g_1^-, \ldots, g_n^-, \Lambda_{\bar{q}}^-) = \frac{p_i^2 \phi_q^3 \phi_{\bar{q}}}{\phi_{i-1}\phi_{i+1}} \frac{1}{[q,1][1,2]\cdots[i-2,i-1][i+1,i+2]\cdots[n,\bar{q}][\bar{q},q]}\,, \tag{4.18}$$

$$A(\Lambda_q^-, g_1^-, \cdots, g_n^-, \Lambda_{\bar{q}}^+) = -\frac{p_i^2 \phi_q \phi_{\bar{q}}^3}{\phi_{i-1}\phi_{i+1}} \frac{1}{[q,1][1,2]\cdots[i-2,i-1][i+1,i+2]\cdots[n,\bar{q}][\bar{q},q]}\,. \tag{4.19}$$

Here the index 0 (whenever it appears) refers to the quark Λ_q.

The proof of eq. (4.18) and eq. (4.19) is similar. Here we present the proof of eq. (4.18) only. As we did earlier in the proof of eq. (4.5), we will again use mathematical induction with the number of gluons n.

To start with, it is easy to check that eq. (4.18) is true for $n = 1$. By assuming that it is true for all $k < n$, we will prove that it is also true for $k = n$.

Because only g_i is off-shell, one can classify all contributing diagrams according to the (3-line) MHV vertices attached to g_i. There are 3 kinds of diagrams, shown in figure 5, figure 6 and figure 7 respectively. By using the assumed result for all less multi-particle amplitudes and eq. (2.7), we can compute the contributions from these diagrams. The contribution from figure 5 is

$$A^1 = \sum_{j=0}^{i-2} \frac{\phi_q^3 \phi_{\bar{q}}}{\phi_j \phi_{i+1}} \frac{1}{[q,1]\cdots[j-1,j][i+1,i+2]\cdots[n,\bar{q}][\bar{q},q]} \times$$

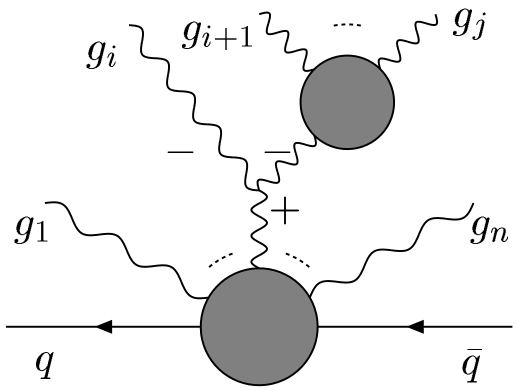

Figure 6: The same kind diagram as in figure 5. Here it is g_{i+1}'s which are disconnected from the fermion line by the 3-gluon MHV vertex.

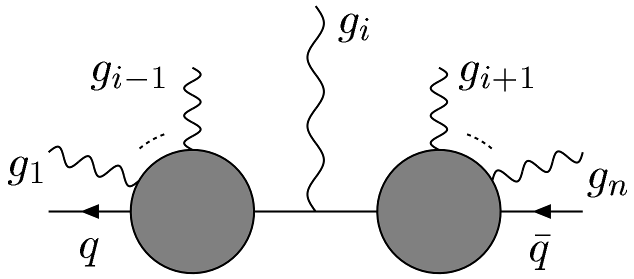

Figure 7: In this diagram, g_i is directly attached to the fermion line.

$$\begin{aligned}
&\times \frac{1}{\phi_{j+1}\phi_{i-1}} \frac{1}{[j+1,j+2]\cdots[i-2,i-1]} \times \langle \lambda_{\tilde{p}}, \lambda_{\tilde{k}} \rangle \\
&= \frac{1}{[q,1][1,2]\cdots[i-2,i-1][i+1,i+2]\cdots[n,\bar{q}][\bar{q},q]} \times \\
&\quad \times \frac{\phi_q^3 \phi_{\bar{q}}}{\phi_{i-1}\phi_{i+1}} \sum_{j=0}^{i-2} \frac{[j,j+1]}{\phi_j \phi_{j+1}} \langle \lambda_{\tilde{p}}, \lambda_{\tilde{k}} \rangle \,, \qquad (4.20)
\end{aligned}$$

where

$$\lambda_{\tilde{p}} = \sum_{l=0}^{j} \lambda_l \phi_l + \sum_{l=i+1}^{n+1} \lambda_l \phi_l \,, \qquad \lambda_{\tilde{k}} = \sum_{l=j+1}^{i-1} \lambda_l \phi_l \,. \qquad (4.21)$$

The contribution from figure 6 can be calculated similarly as above and the result is:

$$\begin{aligned}
A^2 &= \sum_{j=i+1}^{n} \frac{1}{\phi_{i+1}\phi_j} \frac{1}{[i+1,i+2]\cdots[j-1,j]} \frac{\phi_q^3 \phi_{\bar{q}}}{\phi_{i-1}\phi_{j+1}} \times \\
&\quad \times \frac{1}{[q,1]\cdots[i-2,i-1][j+1,j+2]\cdots[n,\bar{q}][\bar{q},q]} \langle \lambda_{\tilde{p}}, \lambda_{\tilde{k}} \rangle \\
&= \frac{1}{[q,1][1,2]\cdots[i-2,i-1][i+1,i+2]\cdots[n,\bar{q}][\bar{q},q]} \times \\
&\quad \times \frac{\phi_q^3 \phi_{\bar{q}}}{\phi_{i-1}\phi_{i+1}} \sum_{j=i+1}^{n} \frac{[j,j+1]}{\phi_j \phi_{j+1}} \langle \lambda_{\tilde{p}}, \lambda_{\tilde{k}} \rangle \,, \qquad (4.22)
\end{aligned}$$

where

$$\lambda_{\tilde{p}} = \sum_{l=i+1}^{j} \lambda_l \phi_l \,, \qquad \lambda_{\tilde{k}} = \sum_{l=0}^{i-1} \lambda_l \phi_l + \sum_{l=j+1}^{n+1} \lambda_l \phi_l \,. \tag{4.23}$$

The last contribution from figure 7 can be calculated by using eq. (4.5) and eq. (4.16) and we have:

$$\begin{aligned} A^3 &= \frac{\phi_q^2}{\phi_{i-1}} \frac{1}{[q,1]\cdots[i-2,i-1]} \frac{1}{\phi_{i+1}} \frac{1}{[i+1,i+2]\cdots[n+1,\bar{q}]} \langle \lambda_{\tilde{p}}, \lambda_{\tilde{k}} \rangle \\ &= \frac{1}{[q,1][1,2]\cdots[i-2,i-1][i+1,i+2]\cdots[n,\bar{q}][\bar{q},q]} \frac{\phi_q^3 \phi_{\bar{q}}}{\phi_{i-1}\phi_{i+1}} \frac{[\bar{q},q]}{\phi_{\bar{q}}\phi_q} \langle \lambda_{\tilde{p}}, \lambda_{\tilde{k}} \rangle \,, \end{aligned} \tag{4.24}$$

where

$$\lambda_{\tilde{p}} = \sum_{l=i+1}^{n+1} \lambda_l \phi_l, \qquad \lambda_{\tilde{k}} = \sum_{l=0}^{i-1} \lambda_l \phi_l. \tag{4.25}$$

By combing all these three contributions together and using the following result:

$$\sum_{j=0}^{i-2} \frac{[j,j+1]}{\phi_j \phi_{j+1}} \langle \lambda_{\tilde{p}}, \lambda_{\tilde{k}} \rangle + \frac{[\bar{q},q]}{\phi_{\bar{q}}\phi_q} \langle \lambda_{\tilde{p}}, \lambda_{\tilde{k}} \rangle + \sum_{j=i+1}^{n} \frac{[j,j+1]}{\phi_j \phi_{j+1}} \langle \lambda_{\tilde{p}}, \lambda_{\tilde{k}} \rangle = p_i^2 \,. \tag{4.26}$$

from eq. (4.12), we have

$$\begin{aligned} A(\Lambda_q^+, g_1^-, \ldots, g_n^-, \Lambda_{\bar{q}}^-) &= \sum_{s=1}^{3} A^s \qquad (4.27) \\ &= \frac{\phi_q^3 \phi_{\bar{q}}}{\phi_{i-1}\phi_{i+1}} \frac{p_i^2}{[q,1][1,2]\cdots[i-2,i-1][i+1,i+2]\cdots[n,\bar{q}][\bar{q},q]} \,. \end{aligned}$$

This ends the proof of eq. (4.18).

Now we begin to compute the googly amplitudes with one quark-anti-quark pair. First we compute the amplitude $A(\Lambda_q^+, g_1^+, \ldots, g_I^-, \ldots, g_n^+, \Lambda_{\bar{q}}^-)$. As we noted in section 3, for googly amplitude, there is only one 4-particle MHV vertex in any contributing diagram. The rest vertices are 3-particle MHV vertices. So we can classify all contributing diagrams according to this 4-particle MHV vertex. Form the 4-particle MHV vertex with one quark-antiquark pair, there are 2 kinds of diagrams. One kind of diagrams is shown figure 8 where the only positive helicity gluon g_I is in the left blob connected to the 4-particle vertex. The other kind of diagrams (which is not shown) can be obtained from figure 8 by interchanging the two gluon lines connected to the 4-particle MHV vertex. The contributions from these two kinds of diagrams are:

$$\begin{aligned} A^1 &= \frac{\phi_q^2 \phi_I^4}{[q,1][1,2]\cdots[n-1,n][n,\bar{q}]} \sum_{i=0}^{I-1} \sum_{j=I}^{n-1} \sum_{k=j+1}^{n} \frac{[i,i+1]}{\phi_i \phi_{i+1}} \frac{[j,j+1]}{\phi_j \phi_{j+1}} \times \\ &\times \frac{[k,k+1]}{\phi_k \phi_{k+1}} \frac{-\langle V_1, V_3 \rangle \langle V_4, V_3 \rangle^3}{\langle V_1, V_2 \rangle \langle V_2, V_3 \rangle \langle V_3, V_4 \rangle \langle V_4, V_1 \rangle} \,, \end{aligned} \tag{4.28}$$

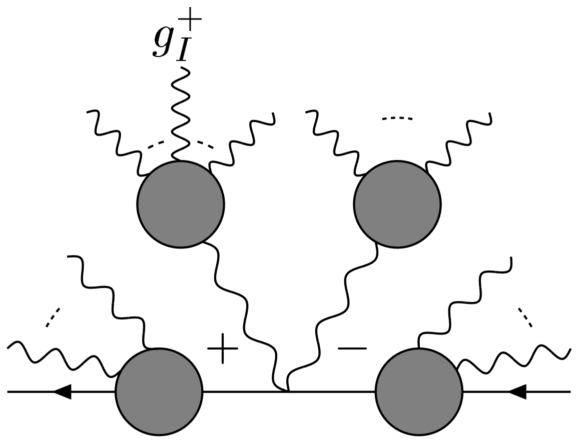

Figure 8: One kind of diagrams from the 4-particle MHV vertex with one quark-antiquark pair.

and

$$A^2 = \frac{\phi_q^2 \phi_I^4}{[q,1][1,2]\cdots[n-1,n][n,\bar{q}]} \sum_{i=0}^{I-2} \sum_{j=i+1}^{I-1} \sum_{k=I}^{n} \frac{[i,i+1]}{\phi_i \phi_{i+1}} \frac{[j,j+1]}{\phi_j \phi_{j+1}} \times$$
$$\times \frac{[k,k+1]}{\phi_k \phi_{k+1}} \frac{-\langle V_1,V_2\rangle \langle V_4,V_2\rangle^3}{\langle V_1,V_2\rangle\langle V_2,V_3\rangle\langle V_3,V_4\rangle\langle V_4,V_1\rangle}, \tag{4.29}$$

respectively. Here

$$V_1 = \sum_{s=0}^{i} \lambda_s \phi_s , \qquad V_2 = \sum_{s=i+1}^{j} \lambda_s \phi_s , \tag{4.30}$$

$$V_3 = \sum_{s=j+1}^{k} \lambda_s \phi_s , \qquad V_4 = \sum_{s=k+1}^{n+1} \lambda_s \phi_s . \tag{4.31}$$

The sum of A^1 and A^2 can be written as follows:

$$A^1 + A^2 = \frac{\phi_q^2 \phi_I^4}{[q,1][1,2]\cdots[n-1,n][n,\bar{q}]} \sum_{i=0}^{I-1} \sum_{k=\max\{I,i+2\}}^{n} \sum_{j=i+1}^{k-1} \frac{[i,i+1]}{\phi_i \phi_{i+1}} \times$$
$$\times \frac{[j,j+1]}{\phi_j \phi_{j+1}} \frac{[k,k+1]}{\phi_k \phi_{k+1}} \frac{-\langle V_1,V_p\rangle \langle V_4,V_p\rangle^3}{\langle V_1,V_2\rangle\langle V_2,V_3\rangle\langle V_3,V_4\rangle\langle V_4,V_1\rangle}, \tag{4.32}$$

where p in eq. (4.32) is the index ($p = 2,3$) which doesn't include $\lambda_I \phi_I$ as defined in eqs. (4.30) and (4.31).

From the 4-gluon MHV vertex, there are 3 kinds of diagrams depending on the position of the only positive helicity gluon g_I. One kind of diagrams is shown in figure 9 where the gluon g_I is in the left blob connected to the 4-gluon MHV vertex. The other 2 kinds of diagrams (which are not shown here) are the same diagrams but with the gluon g_I staying middle blob or in the right blob. The contributions from these diagrams are:

$$\sum_{t=3}^{5} A^t = \frac{\phi_q^3 \phi_{\bar{q}} \phi_I^4}{[q,1][1,2]\cdots[n-1,n][n,\bar{q}][\bar{q},q]} \sum_{i=0}^{I-1} \sum_{l=\max\{I,i+3\}}^{n} \sum_{j=i+1}^{l-2} \sum_{k=j+1}^{l-1} \frac{[i,i+1]}{\phi_i \phi_{i+1}} \times$$

JHEP09(2004)063

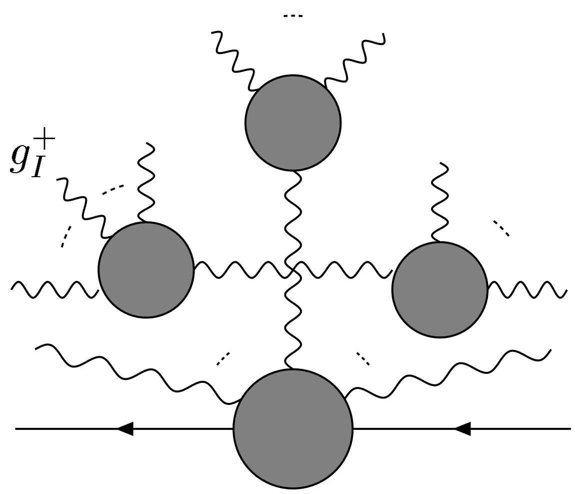

Figure 9: One kind of diagrams from the 4-gluon MHV vertex. Here the only positive helicity gluon g_I is in the far left blob.

$$\times \frac{[j,j+1]}{\phi_j\phi_{j+1}}\,\frac{[k,k+1]}{\phi_k\phi_{k+1}}\,\frac{[l,l+1]}{\phi_l\phi_{l+1}}\,\frac{\langle \tilde V_r,\tilde V_s\rangle^4}{\langle \tilde V_1,\tilde V_2\rangle\langle \tilde V_2,\tilde V_3\rangle\langle \tilde V_3,\tilde V_4\rangle\langle \tilde V_4,\tilde V_1\rangle}\,, \tag{4.33}$$

where

$$\tilde V_1 = \sum_{s=0}^{i}\lambda_s\phi_s + \sum_{s=l+1}^{n+1}\lambda_s\phi_s\,, \qquad \tilde V_2 = \sum_{s=i+1}^{j}\lambda_s\phi_s\,, \tag{4.34}$$

$$\tilde V_3 = \sum_{s=j+1}^{k}\lambda_s\phi_s\,, \qquad \tilde V_4 = \sum_{s=k+1}^{l}\lambda_s\phi_s\,, \tag{4.35}$$

and r and s in eq. (4.33) are the two indexes ($r = 2, 3,\ s = 3, 4,\ r \neq s$) which satisfy that neither $\tilde V_r$ nor $\tilde V_s$ includes $\lambda_I\phi_I$ as defined in eqs. (4.34) and (4.35).

By combining all these contributions, the googly amplitude is

$$A(\Lambda_q^+, g_1^-, \ldots, g_I^+, \ldots, g_n^-, \Lambda_{\bar q}) = \sum_{i=1}^{5} A^i\,. \tag{4.36}$$

In appendix A, we will prove the following identity:

$$\begin{aligned}
&\frac{[\bar q,q]}{\phi_{\bar q}\phi_q}\sum_{i=0}^{I-1}\sum_{k=\max\{I,i+2\}}^{n}\sum_{j=i+1}^{k-1}\frac{[i,i+1]}{\phi_i\phi_{i+1}}\,\frac{[j,j+1]}{\phi_j\phi_{j+1}}\,\frac{[k,k+1]}{\phi_k\phi_{k+1}}\times\\
&\qquad\qquad\times\frac{-\langle V_1,V_p\rangle\langle V_4,V_p\rangle^3}{\langle V_1,V_2\rangle\langle V_2,V_3\rangle\langle V_3,V_4\rangle\langle V_4,V_1\rangle}+\\
&+\sum_{i=1}^{I-1}\sum_{l=\max\{I,i+3\}}^{n}\sum_{j=i+1}^{l-2}\sum_{k=j+1}^{l-1}\frac{[i,i+1]}{\phi_i\phi_{i+1}}\,\frac{[j,j+1]}{\phi_j\phi_{j+1}}\,\frac{[k,k+1]}{\phi_k\phi_{k+1}}\times\\
&\qquad\qquad\times\frac{[l,l+1]}{\phi_l\phi_{l+1}}\,\frac{\langle \tilde V_r,\tilde V_s\rangle^4}{\langle \tilde V_1,\tilde V_2\rangle\langle \tilde V_2,\tilde V_3\rangle\langle \tilde V_3,\tilde V_4\rangle\langle \tilde V_4,\tilde V_1\rangle} = \frac{[q,I]^3[\bar q,I]}{\phi_q^3\phi_{\bar q}\phi_I^4}\,.
\end{aligned} \tag{4.37}$$

By using this identity, we have:

$$A(\Lambda_q^+, g_1^+, \ldots, g_I^-, \ldots, g_n^+, \Lambda_{\bar q}^-) = \frac{[q, I]^3 [\bar q, I]}{[q1][1,2]\cdots[n,\bar q][\bar q, q]}\,, \tag{4.38}$$

which is the expected result for the googly amplitude and it is in agreement with the result obtained by other methods [6].

The calculation for $A(\Lambda_q^-, g_1^+, \cdots, g_I^-, \cdots, g_n^+, \Lambda_{\bar q}^+)$ is similar and the result is eq. (4.2).

5. The 4 quark-anti-quark googly amplitude

In this section we will compute the googly amplitude with two quark-antiquark pairs. Here all the gluons have negative helicity. The googly amplitudes we want to reproduce are given as follows:

$$\begin{aligned} A(\Lambda_{q_1}^{h_1}, l_1, \cdots, l_k, \Lambda_{\bar q_2}^{-h_2}, \Lambda_{q_2}^{h_2}, m_1, \cdots, m_{n-k}, \Lambda_{\bar q_1}^{-h_1}) &= \\ = A_0'(h_1, h_2) \frac{[q_1, \bar q_2]}{[q_1, l_1][l_1, l_2]\cdots[l_k, \bar q_2]} \frac{[q_2, \bar q_1]}{[q_2, m_1][m_1, m_2]\cdots[m_{n-k}, \bar q_1]}\,, \end{aligned} \tag{5.1}$$

where

$$A_0'(+,+) = \frac{[q_1, q_2]^2}{[q_1, \bar q_1][q_2, \bar q_2]}\,, \qquad A_0'(+,-) = -\frac{[q_1, \bar q_2]^2}{[q_1, \bar q_1][q_2, \bar q_2]}\,, \tag{5.2}$$

$$A_0'(-,+) = -\frac{[\bar q_1, q_2]^2}{[q_1, \bar q_1][q_2, \bar q_2]}\,, \qquad A_0'(-,-) = \frac{[\bar q_1, \bar q_2]^2}{[q_1, \bar q_1][q_2, \bar q_2]}\,. \tag{5.3}$$

As discussed in section 3, in all of the diagrams which contribute to the googly amplitudes there is just one MHV vertices with 4 lines. All other vertices are with 3 lines as in the case for the amplitudes with gluons only [12, 13]. So we can classify all contributing diagrams by using this unique 4-particle MHV vertex. Here we only consider the case when $(h_1, h_2) = (+,+)$, other cases are similar.

First we have one kind of diagrams from the 4-gluon MHV vertex as shown in figure 10. Here the two gluons connected to the fermion lines have positive helicity. This contribution is:

$$A^1 = -F_0 \sum_{n_1=0}^{k-2} \sum_{n_2=n_1+1}^{k-1} \sum_{n_3=n_2+1}^{k} \sum_{n_4=0}^{n-k} \tilde F^1(n_1, \cdots, n_4) \frac{\langle V_2^1, V_3^1\rangle^3}{\langle V_1^1, V_2^1\rangle\langle V_3^1, V_4^1\rangle\langle V_4^1, V_1^1\rangle}\,, \tag{5.4}$$

where the index l_0 $(l_{k+1}, m_0, m_{n-k+1})$ refers to q_1 $(\bar q_2, q_2, \bar q_1)$ and F_0 is defined as

$$F_0 = \phi_{q_1}^3 \phi_{\bar q_1} \phi_{q_2}^3 \phi_{\bar q_2} \frac{1}{[\bar q_1, q_1]} \frac{1}{[\bar q_2, q_2]} \prod_{i=0}^{k} \frac{1}{[l_i, l_{i+1}]} \prod_{j=0}^{n-k} \frac{1}{[m_j, m_{j+1}]}\,. \tag{5.5}$$

The other quantities appearing in eq. (5.4) are defined as follows:

$$\tilde F^1(n_1, \cdots, n_4) = \prod_{i=1}^{3} \frac{[l_{n_i}, l_{n_i+1}]}{\phi_{l_{n_i}} \phi_{l_{n_i}+1}} \frac{[m_{n_4}, m_{n_4+1}]}{\phi_{m_{n_4}} \phi_{m_{n_4}+1}}\,, \tag{5.6}$$

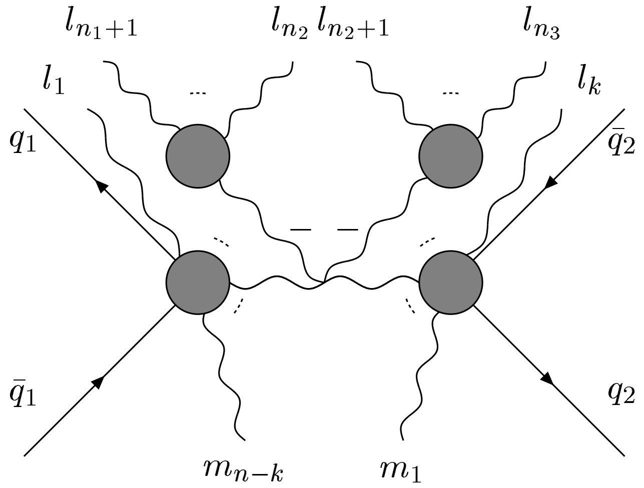

Figure 10: One kind of diagrams from the 4-gluon MHV vertex. The two gluons connected to the fermion lines have positive helicity. This gives contribution A^1. Two more kinds of the same type of diagrams are obtained by permutating 4 internal gluon lines which give contributions $A^{2,3}$.

and

$$V_1^1 = \sum_{s=1}^{n_1} \lambda_{l_s}\phi_{l_s} + \sum_{s=n_4+1}^{n-k} \lambda_{m_s}\phi_{m_s} + \lambda_{q_1}\phi_{q_1} + \lambda_{\bar{q}_1}\phi_{\bar{q}_1}\,, \tag{5.7}$$

$$V_2^1 = \sum_{s=n_1+1}^{n_2} \lambda_{l_s}\phi_{l_s}\,, \tag{5.8}$$

$$V_3^1 = \sum_{s=n_2+1}^{n_3} \lambda_{l_s}\phi_{l_s}\,, \tag{5.9}$$

$$V_4^1 = \sum_{s=n_3+1}^{k} \lambda_{l_s}\phi_{l_s} + \sum_{s=1}^{n_4} \lambda_{m_s}\phi_{m_s} + \lambda_{q_2}\phi_{q_2} + \lambda_{\bar{q}_2}\phi_{\bar{q}_2}\,. \tag{5.10}$$

The other 2 similar contributions are:

$$\begin{aligned} A^2 &= -F_0 \sum_{n_1=0}^{k-1} \sum_{n_2=n_1+1}^{k} \sum_{n_3=0}^{n-k-1} \sum_{n_4=n_3+1}^{n-k} \tilde{F}^2(n_1,\ldots,n_4) \times \\ &\quad \times \frac{\langle V_2^2, V_4^2\rangle^4}{\langle V_1^2, V_2^2\rangle\langle V_2^2, V_3^2\rangle\langle V_3^2, V_4^2\rangle\langle V_4^2, V_1^2\rangle}\,, \end{aligned} \tag{5.11}$$

$$\begin{aligned} A^3 &= -F_0 \sum_{n_1=0}^{k} \sum_{n_2=0}^{n-k-2} \sum_{n_3=n_2+1}^{n-k-1} \sum_{n_4=n_3+1}^{n-k} \tilde{F}^3(n_1,\cdots,n_4) \times \\ &\quad \times \frac{\langle V_3^3, V_4^3\rangle^4}{\langle V_1^3, V_2^3\rangle\langle V_2^3, V_3^3\rangle\langle V_3^3, V_4^3\rangle\langle V_4^3, V_1^3\rangle}\,, \end{aligned} \tag{5.12}$$

where

$$\tilde{F}^2(n_1,\cdots,n_4) = \prod_{i=1}^{2} \frac{[l_{n_i}, l_{n_i+1}]}{\phi_{l_{n_i}}\phi_{l_{n_i}+1}} \prod_{j=3}^{4} \frac{[m_{n_j}, m_{n_j+1}]}{\phi_{m_{n_j}}\phi_{m_{n_j}+1}}\,, \tag{5.13}$$

JHEP09(2004)063

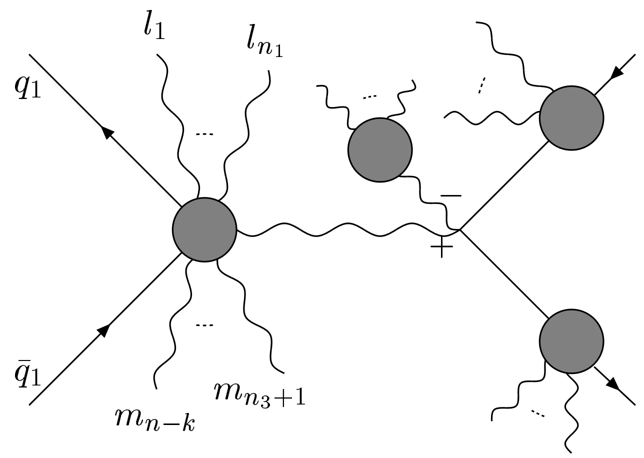

Figure 11: One kind of diagrams from the 4-particle MHV vertex with one quark-antiquark pair. This gives contribution A^4. There are 3 more kinds of the same type of diagrams which give contributions $A^{5,6,7}$.

$$\tilde{F}^3(n_1, \cdots, n_4) = \frac{[l_{n_1}, l_{n_1+1}]}{\phi_{l_{n_1}} \phi_{l_{n_1}+1}} \prod_{j=2}^{4} \frac{[m_{n_j}, m_{n_j+1}]}{\phi_{m_{n_j}} \phi_{m_{n_j}+1}}, \tag{5.14}$$

and

$$V_1^2 = \sum_{s=1}^{n_1} \lambda_{l_s} \phi_{l_s} + \sum_{s=n_4+1}^{n-k} \lambda_{m_s} \phi_{m_s} + \lambda_{q_1} \phi_{q_1} + \lambda_{\bar{q}_1} \phi_{\bar{q}_1} , \tag{5.15}$$

$$V_2^2 = \sum_{s=n_1+1}^{n_2} \lambda_{l_s} \phi_{l_s} , \tag{5.16}$$

$$V_3^2 = \sum_{s=n_2+1}^{k} \lambda_{l_s} \phi_{l_s} + \sum_{s=1}^{n_3} \lambda_{m_s} \phi_{m_s} + \lambda_{q_2} \phi_{q_2} + \lambda_{\bar{q}_2} \phi_{\bar{q}_2} , \tag{5.17}$$

$$V_4^2 = \sum_{s=n_3+1}^{n_4} \lambda_{m_s} \phi_{m_s} , \tag{5.18}$$

$$V_1^3 = \sum_{s=1}^{n_1} \lambda_{l_s} \phi_{l_s} + \sum_{s=n_4+1}^{n-k} \lambda_{m_s} \phi_{m_s} + \lambda_{q_1} \phi_{q_1} + \lambda_{\bar{q}_1} \phi_{\bar{q}_1} , \tag{5.19}$$

$$V_2^3 = \sum_{s=n_1+1}^{k} \lambda_{l_s} \phi_{l_s} + \sum_{s=1}^{n_2} \lambda_{m_s} \phi_{m_s} + \lambda_{q_2} \phi_{q_2} + \lambda_{\bar{q}_2} \phi_{\bar{q}_2} , \tag{5.20}$$

$$V_3^3 = \sum_{s=n_2+1}^{n_3} \lambda_{m_s} \phi_{m_s} , \tag{5.21}$$

$$V_4^3 = \sum_{s=n_3+1}^{n-k} \lambda_{m_s} \phi_{m_s} . \tag{5.22}$$

The second group of contributions are from the diagrams with the 4-particle MHV vertex having one quark-antiquark pair. An example is shown in figure 11. Its contribution

A^4 is given as follows:

$$A^4 = F_0 \sum_{n_1=0}^{k-1} \sum_{n_2=n_1+1}^{k} \sum_{n_3=0}^{n-k} \tilde{F}^4(n_1,n_2,n_3) \frac{\langle V_2^4, V_3^4\rangle^2 \langle V_2^4, V_4^4\rangle}{\langle V_1^4, V_2^4\rangle \langle V_3^4, V_4^4\rangle \langle V_4^4, V_1^4\rangle}\,, \tag{5.23}$$

where

$$\tilde{F}^4(n_1,n_2,n_3) = \prod_{i=1}^{2} \frac{[l_{n_i}, l_{n_i+1}]}{\phi_{l_{n_i}} \phi_{l_{n_i}+1}} \frac{[m_{n_3}, m_{n_3+1}]}{\phi_{m_{n_3}} \phi_{m_{n_3}+1}} \frac{[q_2, \bar{q}_2]}{\phi_{q_2} \phi_{\bar{q}_2}}\,, \tag{5.24}$$

and

$$V_1^4 = \sum_{s=1}^{n_1} \lambda_{l_s} \phi_{l_s} + \sum_{s=n_3+1}^{n-k} \lambda_{m_s} \phi_{m_s} + \lambda_{q_1} \phi_{q_1} + \lambda_{\bar{q}_1} \phi_{\bar{q}_1}\,, \tag{5.25}$$

$$V_2^4 = \sum_{s=n_1+1}^{n_2} \lambda_{l_s} \phi_{l_s}\,, \tag{5.26}$$

$$V_3^4 = \sum_{s=n_2+1}^{k} \lambda_{l_s} \phi_{l_s} + \lambda_{\bar{q}_2} \phi_{\bar{q}_2}\,, \tag{5.27}$$

$$V_4^4 = \sum_{s=1}^{n_3} \lambda_{m_s} \phi_{m_s} + \lambda_{q_2} \phi_{q_2}\,. \tag{5.28}$$

There are 3 more kinds of the same type of diagrams which give contributions $A^{5,6,7}$:

$$A^5 = F_0 \sum_{n_1=0}^{k} \sum_{n_2=0}^{n-k-1} \sum_{n_3=n_2+1}^{n-k} \tilde{F}^5(n_1,n_2,n_3) \frac{\langle V_2^5, V_4^5\rangle^3}{\langle V_1^5, V_2^5\rangle \langle V_2^5, V_3^5\rangle \langle V_4^5, V_1^5\rangle}\,, \tag{5.29}$$

$$A^6 = F_0 \sum_{n_1=0}^{k-1} \sum_{n_2=n_1+1}^{k} \sum_{n_3=0}^{n-k} \tilde{F}^6(n_1,n_2,n_3) \frac{-\langle V_2^6, V_4^6\rangle^3}{\langle V_2^6, V_3^6\rangle \langle V_3^6, V_4^6\rangle \langle V_4^6, V_1^6\rangle}\,, \tag{5.30}$$

$$A^7 = F_0 \sum_{n_1=0}^{k} \sum_{n_2=0}^{n-k-1} \sum_{n_3=n_2+1}^{n-k} \tilde{F}^7(n_1,n_2,n_3) \frac{-\langle V_1^7, V_3^7\rangle \langle V_3^7, V_4^7\rangle^2}{\langle V_1^7, V_2^7\rangle \langle V_2^7, V_3^7\rangle \langle V_4^7, V_1^7\rangle}\,, \tag{5.31}$$

where

$$\tilde{F}^5(n_1,n_2,n_3) = \frac{[l_{n_1}, l_{n_1+1}]}{\phi_{l_{n_1}} \phi_{l_{n_1}+1}} \prod_{j=2}^{3} \frac{[m_{n_j}, m_{n_j+1}]}{\phi_{m_{n_j}} \phi_{m_{n_j}+1}} \frac{[q_2, \bar{q}_2]}{\phi_{q_2} \phi_{\bar{q}_2}}\,, \tag{5.32}$$

$$\tilde{F}^6(n_1,n_2,n_3) = \prod_{i=1}^{2} \frac{[l_{n_i}, l_{n_i+1}]}{\phi_{l_{n_i}} \phi_{l_{n_i}+1}} \frac{[m_{n_3}, m_{n_3+1}]}{\phi_{m_{n_3}} \phi_{m_{n_3}+1}} \frac{[\bar{q}_1, q_1]}{\phi_{\bar{q}_1} \phi_{q_1}}\,, \tag{5.33}$$

$$\tilde{F}^7(n_1,n_2,n_3) = \frac{[l_{n_1}, l_{n_1+1}]}{\phi_{l_{n_1}} \phi_{l_{n_1}+1}} \prod_{j=2}^{3} \frac{[m_{n_j}, m_{n_j+1}]}{\phi_{m_{n_j}} \phi_{m_{n_j}+1}} \frac{[\bar{q}_1, q_1]}{\phi_{\bar{q}_1} \phi_{q_1}}\,, \tag{5.34}$$

and

$$V_1^5 = \sum_{s=1}^{n_1} \lambda_{l_s} \phi_{l_s} + \sum_{s=n_3+1}^{n-k} \lambda_{m_s} \phi_{m_s} + \lambda_{q_1} \phi_{q_1} + \lambda_{\bar{q}_1} \phi_{\bar{q}_1}\,, \tag{5.35}$$

$$V_2^5 = \sum_{s=n_1+1}^{k} \lambda_{l_s}\phi_{l_s} + \lambda_{\bar{q}_2}\phi_{\bar{q}_2}\,, \tag{5.36}$$

$$V_3^5 = \sum_{s=1}^{n_2} \lambda_{m_s}\phi_{l_s} + \lambda_{q_2}\phi_{q_2}\,, \tag{5.37}$$

$$V_4^5 = \sum_{s=n_2+1}^{n_3} \lambda_{m_s}\phi_{m_s}\,, \tag{5.38}$$

$$V_1^6 = \sum_{s=1}^{n_1} \lambda_{l_s}\phi_{l_s} + \lambda_{q_1}\phi_{q_1}\,, \tag{5.39}$$

$$V_2^6 = \sum_{s=n_1+1}^{n_2} \lambda_{l_s}\phi_{l_s}\,, \tag{5.40}$$

$$V_3^6 = \sum_{s=n_2+1}^{k} \lambda_{l_s}\phi_{l_s} + \sum_{s=1}^{n_3} \lambda_{m_s}\phi_{m_s} + \lambda_{q_2}\phi_{q_2} + \lambda_{\bar{q}_2}\phi_{\bar{q}_2}\,, \tag{5.41}$$

$$V_4^6 = \sum_{s=n_3+1}^{n-k} \lambda_{m_s}\phi_{m_s} + \lambda_{\bar{q}_1}\phi_{\bar{q}_1}\,, \tag{5.42}$$

$$V_1^7 = \sum_{s=1}^{n_1} \lambda_{l_s}\phi_{l_s} + \lambda_{q_1}\phi_{q_1}\,, \tag{5.43}$$

$$V_2^7 = \sum_{s=n_1+1}^{k} \lambda_{l_s}\phi_{l_s} + \sum_{s=1}^{n_2} \lambda_{m_s}\phi_{m_s} + \lambda_{q_2}\phi_{q_2} + \lambda_{\bar{q}_2}\phi_{\bar{q}_2}\,, \tag{5.44}$$

$$V_3^7 = \sum_{s=n_2+1}^{n_3} \lambda_{m_s}\phi_{m_s}\,, \tag{5.45}$$

$$V_4^7 = \sum_{s=n_3+1}^{n-k} \lambda_{m_s}\phi_{m_s} + \lambda_{\bar{q}_1}\phi_{\bar{q}_1}\,. \tag{5.46}$$

The last contribution comes from the only kind of contributing diagrams from the 4-fermion MHV vertex as shown in figure 12. This contribution is:

$$A^8 = F_0 \sum_{n_1=0}^{k} \sum_{n_2=0}^{n-k} \tilde{F}^8(n_1,n_2)\, \frac{\langle V_2^8, V_4^8\rangle^2}{\langle V_1^8, V_2^8\rangle\langle V_3^8, V_4^8\rangle}\,, \tag{5.47}$$

where

$$\tilde{F}^8(n_1,n_2) = \frac{[l_{n_1}, l_{n_1+1}]}{\phi_{l_{n_1}}\phi_{l_{n_1}+1}}\,\frac{[m_{n_2}, m_{n_2+1}]}{\phi_{m_{n_2}}\phi_{m_{n_2}+1}}\,\frac{[\bar{q}_1, q_1]}{\phi_{\bar{q}_1}\phi_{q_1}}\,\frac{[q_2, \bar{q}_2]}{\phi_{q_2}\phi_{\bar{q}_2}}\,, \tag{5.48}$$

and

$$V_1^8 = \sum_{s=1}^{n_1} \lambda_{l_s}\phi_{l_s} + \lambda_{q_1}\phi_{q_1}\,, \tag{5.49}$$

$$V_2^8 = \sum_{s=n_1+1}^{k} \lambda_{l_s}\phi_{l_s} + \lambda_{\bar{q}_2}\phi_{\bar{q}_2}\,, \tag{5.50}$$

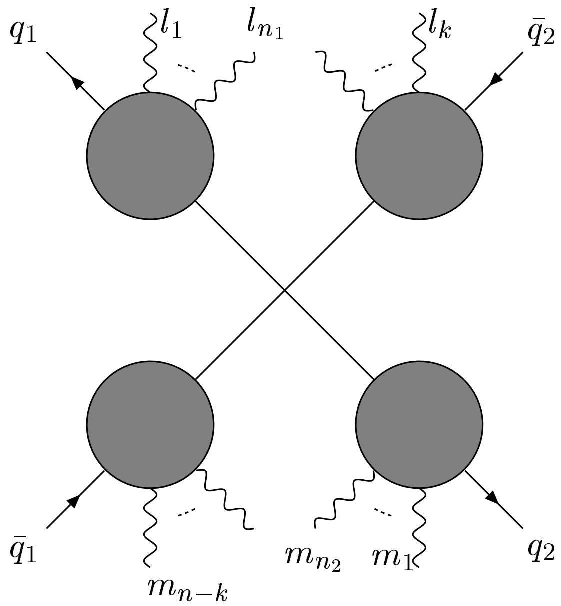

Figure 12: The only kind of contributing diagrams from the 4-fermion MHV vertex. This gives the last contribution A^8.

$$V_3^8 = \sum_{s=1}^{n_2} \lambda_{m_s}\phi_{m_s} + \lambda_{q_2}\phi_{q_2}\,, \tag{5.51}$$

$$V_4^8 = \sum_{s=n_2+1}^{n-k} \lambda_{m_s}\phi_{m_s} + \lambda_{\bar{q}_1}\phi_{\bar{q}_1}\,. \tag{5.52}$$

By summing over all these contributions and using the same method for proving identities as in appendix A, we have:

$$\begin{aligned}\sum_{i=1}^{8} A^i &= \frac{[q_1,q_2]^2}{[q_1,\bar{q}_1][q_2,\bar{q}_2]}\frac{[q_1,\bar{q}_2]}{[q_1,l_1][l_1,l_2]\cdots[l_k,\bar{q}_2]}\times \\ &\quad\times\frac{[q_2,\bar{q}_1]}{[q_2,m_1][m_1,m_2]\cdots[m_{n-k},\bar{q}_1]}\,. \end{aligned} \tag{5.53}$$

This is the correct result for the googly amplitude with two quark-antiquark pairs and n gluons.

6. The gluino googly amplitudes

In this last section we will compute the googly amplitudes with gluinos. As in the case without fermions [12], we first calculate the off shell amplitudes with all gluons having negative helicity. From eq. (3.9) with $n_+ = 0$ and $m = 1$, only n_3 and n_1^{2f} could be non-vanishing. So all contributing diagrams are composed of 3-line MHV vertices. Again we will find that these off shell amplitudes are proportional to p_i^2 (p_i is the off-shell momentum) and they vanish when p_i is also on shell ($p_i^2 = 0$). We note that the vanishing for the

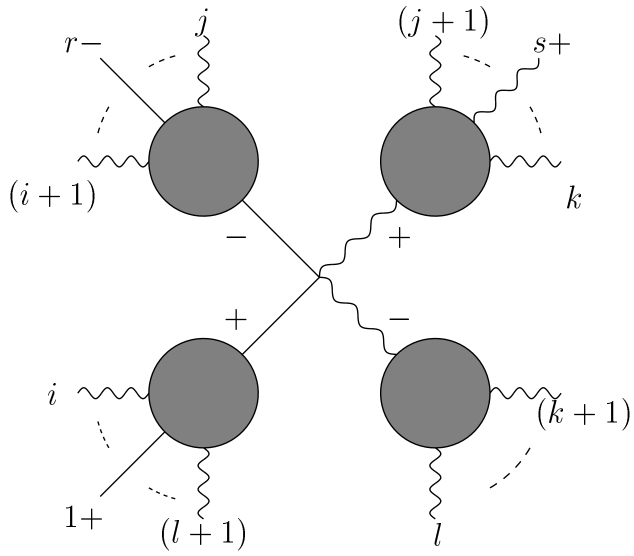

Figure 13: The decomposition for the googly amplitude with fermions when $1 \leq i \leq r-1, r \leq j \leq s-1, s \leq k < l \leq n$.

4-particle on-shell amplitudes were also obtained in [22] and the googly amplitudes with 5 particles were also computed.

We relabel the off shell particle to be the first particle. The helicity of this off-shell particle can be 1/2, $-1/2$ (the gluino) or -1 (the gluon). The corresponding off shell amplitudes are:

$$A_n(\Lambda_1^+, g_2^-, \ldots, \Lambda_s^-, \ldots, g_n^-) = \frac{\phi_s}{\phi_2\phi_n}\frac{p_1^2}{\prod_{i=2}^{n-1}[i, i+1]}, \tag{6.1}$$

$$A_n(\Lambda_1^-, g_2^-, \ldots, \Lambda_r^+, \ldots, g_n^-) = -\frac{\phi_r^3}{\phi_2\phi_n}\frac{p_1^2}{\prod_{i=2}^{n-1}[i, i+1]}, \tag{6.2}$$

and

$$A_n(g_1^-, \ldots, \Lambda_r^+, \ldots, \Lambda_s^-, \ldots, g_n^-) = \frac{\phi_r^3\phi_s}{\phi_2\phi_n}\frac{p_1^2}{\prod_{j=2}^{n-1}[i, i+1]}. \tag{6.3}$$

respectively. The proof of the above formulas is quite similar to the proof for eqs. (2.7) and (2.8) given in [12]. We will not repeat the proof here.

Now we begin to calculate the googly amplitudes. First we consider the googly amplitudes with 2 gluinos: $A_n(\Lambda_1^+, g_2^-, \ldots, \Lambda_r^-, \ldots, g_s^+, \ldots, g_n^-)$. We will consider the case for $1 < r < s \neq n$ only. The rest cases are similar. As we proved in section 3, there is only one vertex with 4 lines. So we can again use the same organization of all the contributing diagrams as in the computation of googly amplitude with one quark-antiquark pair in section 4.

All contributing diagrams are shown in the figures from figure 13 to figure 18. (The signs in the middle of these figures are understood to be the helicities of the particles which belong to the 4-line vertices.) The various contributions corresponding to the figure 13,

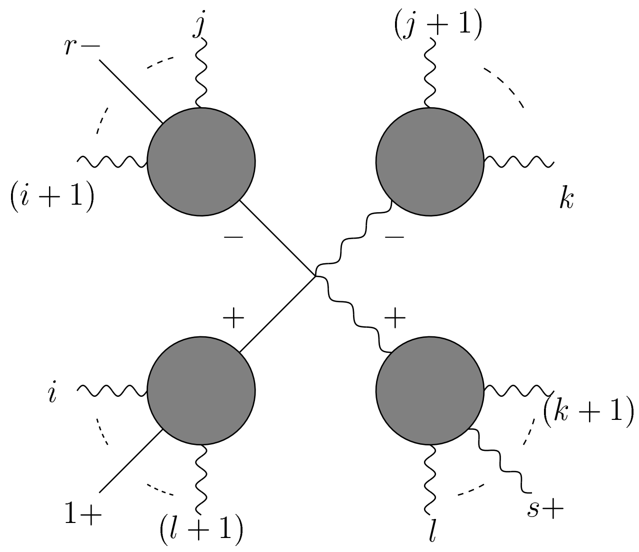

Figure 14: The decomposition for the googly amplitude with fermions when $1 \le i \le r-1, r \le j < k \le s-1, s \le l \le n$.

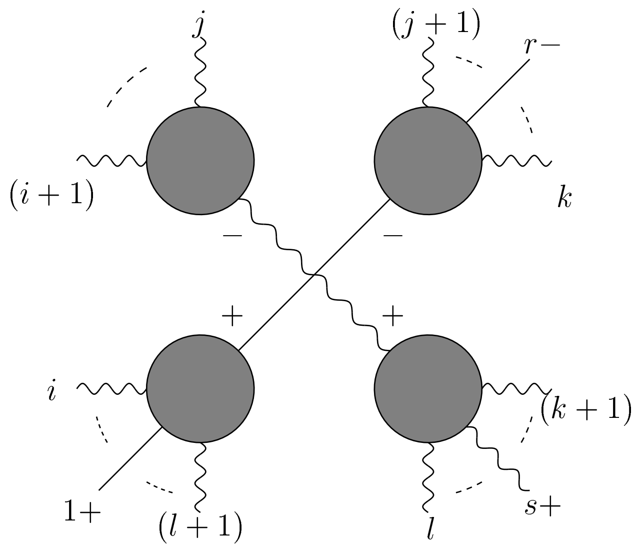

Figure 15: The decomposition for the googly amplitude with fermions when $1 \le i < j \le r-1, r \le k \le s-1, s \le l \le n$.

figure 14, figure 15, figure 16, figure 17 and figure 18 are

$$\begin{aligned} A_n^1 &= \frac{\phi_1^3 \phi_r \phi_s^4}{\prod_{s=1}^n [s,s+1]} \sum_{i=1}^{r-1} \sum_{j=r}^{s-1} \sum_{k=s}^{n-1} \sum_{l=k+1}^{n} \frac{[i,i+1]}{\phi_i \phi_{i+1}} \frac{[k,k+1]}{\phi_k \phi_{k+1}} \times \\ &\quad \times \frac{[j,j+1]}{\phi_j \phi_{j+1}} \frac{[l,l+1]}{\phi_l \phi_{l+1}} \frac{-\langle V_4, V_2\rangle^3}{\langle V_1, V_2\rangle \langle V_2, V_3\rangle \langle V_3, V_4\rangle}, \\ A_n^2 &= \frac{\phi_1^3 \phi_r \phi_s^4}{\prod_{s=1}^n [s,s+1]} \sum_{i=1}^{r-1} \sum_{j=r}^{s-2} \sum_{k=j+1}^{s-1} \sum_{l=s}^{n} \frac{[i,i+1]}{\phi_i \phi_{i+1}} \frac{[k,k+1]}{\phi_k \phi_{k+1}} \times \end{aligned} \tag{6.4}$$

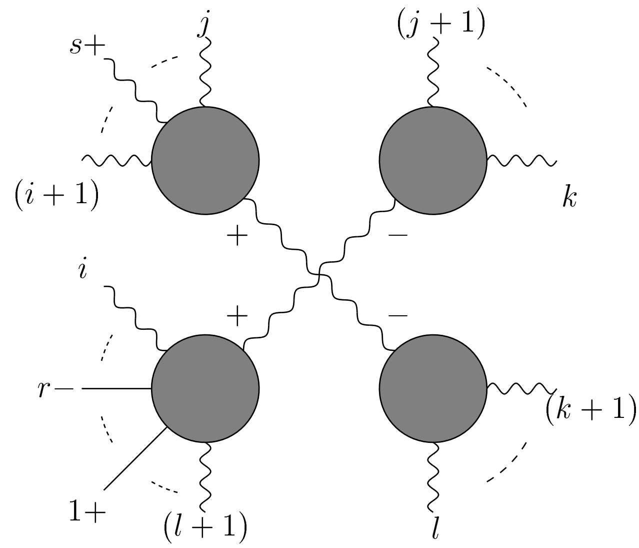

Figure 16: The decomposition for the googly amplitude with fermions when $r \leq i \leq s-1, s \leq j < k < l \leq n$.

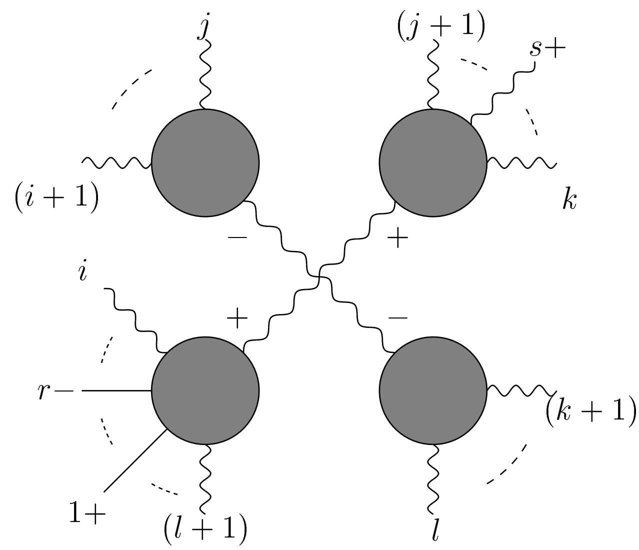

Figure 17: The decomposition for the googly amplitude with fermions when $r \leq i < j \leq s-1, s \leq k < l \leq n$.

$$\times \frac{[j,j+1]}{\phi_j \phi_{j+1}} \frac{[l,l+1]}{\phi_l \phi_{l+1}} \frac{\langle V_3, V_2\rangle^2 \langle V_3, V_1\rangle}{\langle V_1, V_2\rangle \langle V_3, V_4\rangle \langle V_4, V_1\rangle}\,, \tag{6.5}$$

$$A_n^3 = \frac{\phi_1^3 \phi_r \phi_s^4}{\prod_{s=1}^n [s,s+1]} \sum_{i=1}^{r-2} \sum_{j=i+1}^{r-1} \sum_{k=r}^{s-1} \sum_{l=s}^{n} \frac{[i,i+1]}{\phi_i \phi_{i+1}} \frac{[k,k+1]}{\phi_k \phi_{k+1}} \times$$

$$\times \frac{[j,j+1]}{\phi_j \phi_{j+1}} \frac{[l,l+1]}{\phi_l \phi_{l+1}} \frac{\langle V_2, V_3\rangle^2}{\langle V_3, V_4\rangle \langle V_4, V_1\rangle}\,, \tag{6.6}$$

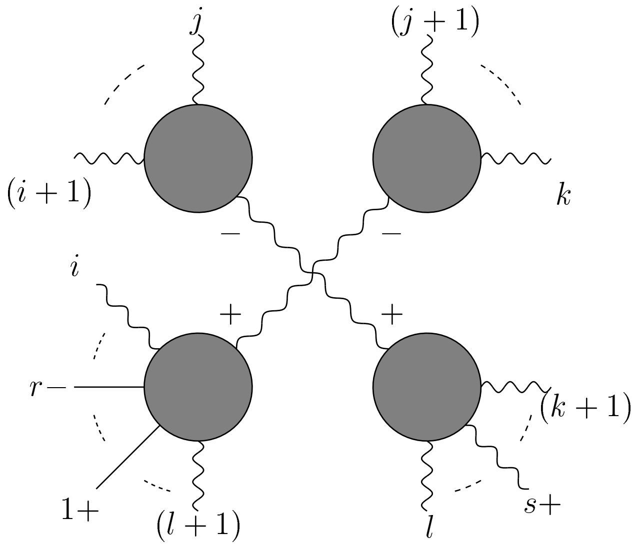

Figure 18: The decomposition for the googly amplitude with fermions when $r \le i < j < k \le s-1, s < l \le n$.

$$A_n^4 = \frac{\phi_1^3\phi_r\phi_s^4}{\prod_{s=1}^n [s,s+1]} \sum_{i=r}^{s-1}\sum_{j=s}^{n-2}\sum_{k=j+1}^{n-1}\sum_{l=k+1}^{n} \frac{[i,i+1]}{\phi_i\phi_{i+1}}\frac{[k,k+1]}{\phi_k\phi_{k+1}} \times$$
$$\times \frac{[j,j+1]}{\phi_j\phi_{j+1}}\frac{[l,l+1]}{\phi_l\phi_{l+1}}\frac{\langle V_3,V_4\rangle^3}{\langle V_1,V_2\rangle\langle V_2,V_3\rangle\langle V_4,V_1\rangle}\,, \tag{6.7}$$

$$A_n^5 = \frac{\phi_1^3\phi_r\phi_s^4}{\prod_{s=1}^n [s,s+1]} \sum_{i=r}^{s-2}\sum_{j=i+1}^{s-1}\sum_{k=s}^{n-1}\sum_{l=k+1}^{n} \frac{[i,i+1]}{\phi_i\phi_{i+1}}\frac{[k,k+1]}{\phi_k\phi_{k+1}} \times$$
$$\times \frac{[j,j+1]}{\phi_j\phi_{j+1}}\frac{[l,l+1]}{\phi_l\phi_{l+1}}\frac{\langle V_2,V_4\rangle^4}{\langle V_1,V_2\rangle\langle V_2,V_3\rangle\langle V_3,V_4\rangle\langle V_4,V_1\rangle}\,, \tag{6.8}$$

and

$$A_n^6 = \frac{\phi_1^3\phi_r\phi_s^4}{\prod_{s=1}^n [s,s+1]} \sum_{i=r}^{s-3}\sum_{j=i+1}^{s-2}\sum_{k=j+1}^{s-1}\sum_{l=s}^{n} \frac{[i,i+1]}{\phi_i\phi_{i+1}}\frac{[k,k+1]}{\phi_k\phi_{k+1}} \times$$
$$\times \frac{[j,j+1]}{\phi_j\phi_{j+1}}\frac{[l,l+1]}{\phi_l\phi_{l+1}}\frac{\langle V_2,V_3\rangle^3}{\langle V_1,V_2\rangle\langle V_3,V_4\rangle\langle V_4,V_1\rangle}\,, \tag{6.9}$$

where[2]

$$V_1 = \sum_{s=l+1}^{n+i}\lambda_s\phi_s\,, \qquad V_2 = \sum_{s=i+1}^{j}\lambda_s\phi_s\,, \tag{6.10}$$

$$V_3 = \sum_{s=j+1}^{k}\lambda_s\phi_s\,, \qquad V_4 = \sum_{s=k+1}^{l}\lambda_s\phi_s\,. \tag{6.11}$$

[2]Here and in the following, the summation like $\sum_{s=l+1}^{n+i}$ is understood as $\sum_{s=l+1}^{n}+\sum_{s=1}^{i}$.

The sum of $A_n^{1,2,3}$ can be written as

$$A_n^1 + A_n^2 + A_n^3 = \frac{\phi_1^3 \phi_r \phi_s^4}{\prod_{s=1}^n [s, s+1]} \sum_{i=1}^{r-1} \sum_{l=\max\{s,i+3\}}^{n} \sum_{j=i+1}^{l-2} \sum_{k=j+1}^{l-1} \times \\ \times \frac{[i, i+1]}{\phi_i \phi_{i+1}} \frac{[k, k+1]}{\phi_k \phi_{k+1}} \frac{[j, j+1]}{\phi_j \phi_{j+1}} \frac{[l, l+1]}{\phi_l \phi_{l+1}} \times \\ \times \frac{-\langle V_a, V_b \rangle^3 \langle V_a, V_1 \rangle}{\langle V_1, V_2 \rangle \langle V_2, V_3 \rangle \langle V_3, V_4 \rangle \langle V_4, V_1 \rangle}, \tag{6.12}$$

where a is an index ($a = 3, 4$) which satisfies the condition that V_a includes neither $\lambda_r \phi_r$ nor $\lambda_s \phi_s$ as defined in eqs. (6.10) and (6.11), and b is an index ($b = 2, 3, b < a$) which satisfies the condition that V_b includes $\lambda_r \phi_r$ as defined in eqs. (6.10) and (6.11).

The sum of $A_n^{4,5,6}$ can be similarly organized and we have:

$$A_n^4 + A_n^5 + A_n^6 = \frac{\phi_1^3 \phi_r \phi_s^4}{\prod_{s=1}^n [s, s+1]} \sum_{i=r}^{s-1} \sum_{l=\max\{s,i+3\}}^{n} \sum_{j=i+1}^{l-2} \sum_{k=j+1}^{l-1} \times \\ \times \frac{[i, i+1]}{\phi_i \phi_{i+1}} \frac{[k, k+1]}{\phi_k \phi_{k+1}} \frac{[j, j+1]}{\phi_j \phi_{j+1}} \frac{[l, l+1]}{\phi_l \phi_{l+1}} \times \\ \times \frac{\langle V_p, V_q \rangle^4}{\langle V_1, V_2 \rangle \langle V_2, V_3 \rangle \langle V_3, V_4 \rangle \langle V_4, V_1 \rangle}, \tag{6.13}$$

where p and q are the two indexes ($p = 2, 3,\ q = 3, 4,\ p \neq q$) which satisfy that neither V_p nor V_q includes $\lambda_r \phi_r$ as defined in eqs. (6.10) and (6.11).

In order to obtain the correct result for the googly amplitude, all we need is to prove the following identity:

$$\sum_{i=1}^{r-1} \sum_{l=\max\{s,i+3\}}^{n} \sum_{j=i+1}^{l-2} \sum_{k=j+1}^{l-1} \frac{[i, i+1]}{\phi_i \phi_{i+1}} \frac{[j, j+1]}{\phi_j \phi_{j+1}} \times \\ \times \frac{[k, k+1]}{\phi_k \phi_{k+1}} \frac{[l, l+1]}{\phi_l \phi_{l+1}} \frac{-\langle V_a, V_b \rangle^3 \langle V_a, V_1 \rangle}{\langle V_1, V_2 \rangle \langle V_2, V_3 \rangle \langle V_3, V_4 \rangle \langle V_4, V_1 \rangle} + \\ + \sum_{i=r}^{s-1} \sum_{l=\max\{s,i+3\}}^{n} \sum_{j=i+1}^{l-2} \sum_{k=j+1}^{l-1} \frac{[i, i+1]}{\phi_i \phi_{i+1}} \frac{[j, j+1]}{\phi_j \phi_{j+1}} \times \\ \times \frac{[k, k+1]}{\phi_k \phi_{k+1}} \frac{[l, l+1]}{\phi_l \phi_{l+1}} \frac{\langle V_p, V_q \rangle^4}{\langle V_1, V_2 \rangle \langle V_2, V_3 \rangle \langle V_3, V_4 \rangle \langle V_4, V_1 \rangle} = \frac{[s, 1]^3 [s, r]}{\phi_s^4 \phi_1^3 \phi_r}. \tag{6.14}$$

This is indeed the case and one can prove it by using the same method used in [12, 13] and appendix A. We will not give the proof here.

By using this identity , we have:

$$A_n = \sum_{i=1}^{6} A_n^i = [s, 1]^3 [s, r] / (\prod_{i=1}^{n} [i, i+1]), \tag{6.15}$$

which is the right result for the googly amplitude with 2 gluinos.

As we did in section 5, we can similarly calculate the googly amplitudes with 4-gluinos (all gluons then have negative helicity) and the result is:

$$A(g_1^-, \ldots, \Lambda_r^+, \ldots, \Lambda_s^+, \ldots, \Lambda_p^-, \ldots, \Lambda_q^-, \ldots, g_n^-) = -\frac{[p,q]^3 [r,s]}{\prod_{i=1}^n [i, i+1]}\,, \tag{6.16}$$

as expected. We will not present any details here.

Acknowledgments

We would like to thank Zhe Chang, Bin Chen, Han-Ying Guo, Miao Li, Jian-Xin Lu, Jian-Ping Ma, Ke Wu and Yong-Shi Wu for discussions. Jun-Bao Wu would like to thank Qiang Li for help on drawing the figures. Chuan-Jie Zhu would like to thank Jian-Xin Lu and the hospitality at the Interdisciplinary Center for Theoretical Study, University of Science and Technology of China where part of this work was done.

A. The proof of eq. (4.37)

In this appendix, we will prove eq. (4.37). As in [12, 13], we can use an SL$(2,\mathbf{C})$ transformation and a rescaling of $\tilde{\eta}$ to choose $\tilde{\eta}^1 = 0$ and $\tilde{\eta}^2 = 1$. We then do a rescaling of $\tilde{\lambda}_{i1}$ by $\tilde{\lambda}_{i2}$, i.e. by defining $\varphi_i = \frac{\tilde{\lambda}_{i1}}{\tilde{\lambda}_{i2}}$, and also do a rescaling of λ_{ia} by $1/\tilde{\lambda}_{i2}$. After using these redefinitions, eq. (4.37) becomes:

$$\begin{aligned}
F_f(\varphi_i) \equiv\ & (\varphi_{\bar{q}} - \varphi_q) \sum_{i=0}^{I-1} \sum_{k=\max\{I,i+2\}}^{n} \sum_{j=i+1}^{k-1} (\varphi_i - \varphi_{i+1})(\varphi_j - \varphi_{j+1}) \times \\
& \times (\varphi_k - \varphi_{k+1}) \frac{-\langle V_1, V_p\rangle \langle V_4, V_p\rangle^3}{\langle V_1, V_2\rangle \langle V_2, V_3\rangle \langle V_3, V_4\rangle \langle V_4, V_1\rangle} + \\
& + \sum_{i=0}^{I-1} \sum_{l=\max\{I,i+3\}}^{n} \sum_{j=i+1}^{l-2} \sum_{k=j+1}^{l-1} (\varphi_i - \varphi_{i+1})(\varphi_j - \varphi_{j+1})(\varphi_k - \varphi_{k+1})(\varphi_l - \varphi_{l+1}) \times \\
& \times \frac{\langle \tilde{V}_r, \tilde{V}_s\rangle^4}{\langle \tilde{V}_1, \tilde{V}_2\rangle \langle \tilde{V}_2, \tilde{V}_3\rangle \langle \tilde{V}_3, \tilde{V}_4\rangle \langle \tilde{V}_4, \tilde{V}_1\rangle} = (\varphi_q - \varphi_I)^3 (\varphi_{\bar{q}} - \varphi_I)\,,
\end{aligned} \tag{A.1}$$

where

$$V_1 = \sum_{s=0}^{i} \lambda_s\,, \qquad V_2 = \sum_{s=i+1}^{j} \lambda_s\,, \tag{A.2}$$

$$V_3 = \sum_{s=j+1}^{k} \lambda_s\,, \qquad V_4 = \sum_{s=k+1}^{n+1} \lambda_s\,, \tag{A.3}$$

$$\tilde{V}_1 = \sum_{s=0}^{i} \lambda_s + \sum_{s=l+1}^{n+1} \lambda_s\,, \qquad \tilde{V}_2 = \sum_{s=i+1}^{j} \lambda_s\,, \tag{A.4}$$

$$\tilde{V}_3 = \sum_{s=j+1}^{k} \lambda_s\,, \qquad \tilde{V}_4 = \sum_{s=k+1}^{l} \lambda_s\,. \tag{A.5}$$

JHEP09(2004)063

There are also two constraints:

$$\sum_{i=1}^{4} V_i = \sum_{i=1}^{4} \tilde{V}_i = \sum_{l=0}^{n+1} \lambda_l = 0\,, \tag{A.6}$$

$$\sum_{l=0}^{n+1} \lambda_i \varphi_i = 0\,, \tag{A.7}$$

from momentum conservation.

From eq. (A.6) and eq. (A.7) we can solve λ_0 and λ_I in terms of the rest λ_i and all φ_j's as:

$$\lambda_0 = -\sum_{1\le j\le n+1, j\ne I} \frac{\varphi_j - \varphi_I}{\varphi_0 - \varphi_I}\lambda_j\,, \tag{A.8}$$

$$\lambda_I = \sum_{1\le j\le n+1, j\ne I} \frac{\varphi_j - \varphi_0}{\varphi_0 - \varphi_I}\lambda_j\,. \tag{A.9}$$

By using this solution, we can consider $F_f(\varphi_i)$ as a function of λ_j $(1 \le j \le n+1,\, j \ne I)$ and all φ_j's $(0 \le j \le n+1)$.

To prove eq. (A.1), we first prove that $F_f(\varphi_i)$ is independent of φ_j for $1 \le j \le n, j \ne I$ and $F_f(\varphi_i)$ depends on φ_{n+1} linearly by analyzing the pole terms.

First we will show that when $\varphi_s \to \infty(1 \le s \le n, s \ne I)$ the pole terms are vanishing in $F_f(\varphi)$ and when $\varphi_{n+1} \to \infty$, $F_f(\varphi)$ will grow as φ_{n+1} at most.

When $\varphi_s \to \infty(1 \le s \le n, s \ne I)$, λ_1 and λ_I will grow as φ_s, and other λ_i's don't depend on φ_s. One can easily find that the pole terms in the second sum in eq. (A.1),

$$\begin{aligned}&\sum_{i=0}^{I-1} \sum_{l=\max\{I,i+3\}}^{n} \sum_{j=i+1}^{l-2} \sum_{k=j+1}^{l-1} (\varphi_i - \varphi_{i+1})(\varphi_j - \varphi_{j+1})(\varphi_k - \varphi_{k+1})(\varphi_l - \varphi_{l+1}) \times \\ &\times \frac{\langle \tilde{V}_r, \tilde{V}_s\rangle^4}{\langle \tilde{V}_1, \tilde{V}_2\rangle\langle \tilde{V}_2, \tilde{V}_3\rangle\langle \tilde{V}_3, \tilde{V}_4\rangle\langle \tilde{V}_4, \tilde{V}_1\rangle}\,,\end{aligned} \tag{A.10}$$

are vanishing as in [12, 13].

As to the terms in the first sum in eq. (A.1), it is easy to see that V_1 and V_q $(q = 2, 3, q \ne p)$ will grow as φ_s.

If none of the V_i's $(i = 1, \ldots, 4)$ includes only the term λ_s, the factor $F(i, j, k) \equiv (\varphi_{\bar{q}} - \varphi_q)(\varphi_i - \varphi_{i+1})(\varphi_j - \varphi_{j+1})(\varphi_k - \varphi_{k+1})$ grows as φ_s at most, $-\langle V_1, V_p\rangle\langle V_4, V_p\rangle^3$ grow as φ_s at most, and $\langle V_1, V_2\rangle\langle V_2, V_3\rangle\langle V_3, V_4\rangle\langle V_4, V_1\rangle$ grows as φ_s^3 at least. So the pole terms are vanishing.

The other case is when one of the V_i's includes only the term λ_s (this i must be p), then $F(i, j, k)$ grows as φ_s^2 at most, $-\langle V_1, V_p\rangle\langle V_4, V_p\rangle^3$ is independent of φ_s, and $\langle V_1, V_2\rangle\langle V_2, V_3\rangle\langle V_3, V_4\rangle\langle V_4, V_1\rangle$ grows as φ_s^2 at least. So no terms will tend to infinity.

So when $\varphi_s \to \infty$ $(1 \le s \le n, s \ne I)$ the pole terms are vanishing.

When $\varphi_{n+1} \to \infty$, λ_1 and λ_I will grow as φ_{n+1}, and other λ_i's don't depend on φ_{n+1}. One can find that the pole terms in eq. (A.10), are still vanishing.

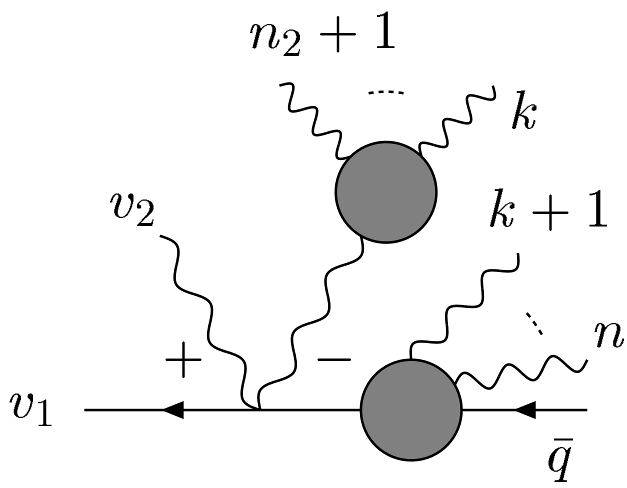

Figure 19: Contributing diagrams from the vanishing of $\langle V_1, V_2\rangle$. The other 2 contributing diagrams from the vanishing of $\langle V_3, V_4\rangle$ or $\langle V_4, V_1\rangle$ are similar and are not shown.

As to the terms in the first sum in eq. (A.1), it is easy to see that V_1 and V_q ($q = 2, 3, q \neq p$) will grow as φ_{n+1}.

If none of the V_i's ($i = 1, \ldots, 4$) includes only the term φ_{n+1}, the factor $F(i,j,k)$ grows as φ_{n+1} at most, $-\langle V_1, V_p\rangle\langle V_4, V_p\rangle^3$ grow as φ_{n+1} at most, and $\langle V_1, V_2\rangle\langle V_2, V_3\rangle\langle V_3, V_4\rangle\langle V_4, V_1\rangle$ grows as φ^3_{n+1} at least. So the pole terms are vanishing.

The other case is when one of the V_i's includes only the term φ_{n+1} (this i must be 4), then $F(i,j,k)$ grows as φ^2_{n+1} at most, $-\langle V_1, V_p\rangle\langle V_4, V_p\rangle^3$ grows as φ_{n+1} as most, and $\langle V_1, V_2\rangle\langle V_2, V_3\rangle\langle V_3, V_4\rangle\langle V_4, V_1\rangle$ grows as φ^2_{n+1} at least. So every term grows as φ_{n+1} at most.

So when $\varphi_{n+1} \to \infty$, $F_f(\varphi)$ will grow as φ_{n+1} at most.

Now we will show that the finite poles terms will vanishing.

There are pole terms if any factor of $\langle V_1, V_2\rangle\langle V_2, V_3\rangle\langle V_3, V_4\rangle\langle V_4, V_1\rangle$ is vanishing or any factor of $\langle \tilde{V}_1, \tilde{V}_2\rangle\langle \tilde{V}_2, \tilde{V}_3\rangle\langle \tilde{V}_3, \tilde{V}_4\rangle\langle \tilde{V}_4, \tilde{V}_1\rangle$ is vanishing. Let us consider first the vanishing of $\langle V_1, V_2\rangle$. We denote this set of V_1 and V_2 as v_1 and v_2:

$$v_1 = \lambda_0 + \cdots + \lambda_{n_1}\,, \qquad v_2 = \lambda_{n_1+1} + \cdots + \lambda_{n_2}\,. \tag{A.11}$$

If $n_2 < I$, then every individual term will not tend to infinity. So we turn to consider the case when $n_1 < I \leq n_2$. As one can see from figure 19 that there are contributions from summing over k and fixing $i = n_1$ and $j = n_2$. The residues (ignoring an overall factor $(\varphi_{\bar{q}} - \varphi_q)(\varphi_{n_1} - \varphi_{n_1+1})(\varphi_{n_2} - \varphi_{n_2+1})$) for these pole terms are:

$$C_1 = \sum_{k=n_2+1}^{n} (\varphi_k - \varphi_{k+1}) \frac{-\langle V_3, V_4\rangle^2\langle V_3, v_1\rangle}{\langle v_2, V_3\rangle\langle V_4, v_1\rangle}\,. \tag{A.12}$$

Because $\langle v_1, v_2\rangle = 0$, v_1 and v_2 are linearly dependent, we can assume that $v_i = \alpha_i v_0, i = 1, 2$, for some α_i and v_0.

Using this result and $v_1 + v_2 + V_3 + V_4 = 0$, we have

$$C_1 = \frac{(\alpha_1+\alpha_2)^2}{\alpha_2} \sum_{k=n_2+1}^{n} (\varphi_k - \varphi_{k+1})\langle v_0, V_3\rangle \,. \tag{A.13}$$

Similar pole terms can also be obtained from the vanishing of the factor $\langle V_3, V_4\rangle$ by setting $V_3 = v_2$ and $V_4 = -(v_1+v_2)$. This gives the following contribution:

$$\begin{aligned} C_2 &= \sum_{i=0}^{n_1-1} (\varphi_i - \varphi_{i+1}) \frac{\langle V_2, -v_1 - v_2\rangle^3}{\langle V_2, v_2\rangle\langle -(v_1+v_2), V_1\rangle} \\ &= \frac{(\alpha_1+\alpha_2)^2}{\alpha_2} \sum_{i=0}^{n_1-1} (\varphi_i - \varphi_{i+1})\,\langle v_0, V_1\rangle \,. \end{aligned} \tag{A.14}$$

The contribution obtained from the vanishing of the factor $\langle V_4, V_1\rangle$ by setting $V_1 = v_1$ and $V_4 = -(v_1+v_2)$ is:

$$C_3 = \frac{(\alpha_1+\alpha_2)^2}{\alpha_2} \sum_{j=n_1+1}^{n_2-1} (\varphi_j - \varphi_{j+1})\,\langle v_0, V_1\rangle \,. \tag{A.15}$$

Using the same algebraic manipulation as given in [12, 13], one finds that $C_1+C_2+C_3 = 0$.

Now we consider the case when $\langle \tilde V_1, \tilde V_2\rangle$ vanishing. We denote this set of $\tilde V_1$ and $\tilde V_2$ as $\tilde v_1$ and $\tilde v_2$:

$$\tilde v_1 = \lambda_{n_3+1} + \cdots + \lambda_n + \lambda_{\bar q} + \lambda_q + \lambda_1 + \cdots + \lambda_{n_1} \,, \tag{A.16}$$

$$\tilde v_2 = \lambda_{n_1+1} + \cdots + \lambda_{n_2} \,. \tag{A.17}$$

We consider the case for $n_1 + 1 \le I \le n_2$, the case for $n_2 + 1 \le I \le n_3$ can be treated similarly. The rest cases are more easier as in [13]. Because $\langle \tilde v_1, \tilde v_2\rangle = 0$, we can assume that $\tilde v_i = \alpha_i \tilde v_0$, $i = 1, 2$.

The residues corresponding to figure 20 from the vanishing of $\langle \tilde V_1, \tilde V_2\rangle$ is:

$$\begin{aligned} \tilde C_1 &= \sum_{k=n_2+1}^{n_3-1} (\varphi_k - \varphi_{k+1}) \frac{\langle \tilde V_3, -\tilde v_1 - \tilde v_2 - \tilde V_3\rangle^3}{\langle \tilde v_2, \tilde V_3\rangle\langle -\tilde v_1 - \tilde v_2 - \tilde V_3, \tilde v_1\rangle} \\ &= \frac{(\alpha_1+\alpha_2)^3}{\alpha_1\alpha_2} \sum_{k=n_2+1}^{n_3-1} (\varphi_k - \varphi_{k+1})\,\langle \tilde v_0, \tilde V_3\rangle \,, \end{aligned} \tag{A.18}$$

by omitting an overall factor $(\varphi_{n_3} - \varphi_{n_3+1})(\varphi_{n_1} - \varphi_{n_1+1})(\varphi_{n_2} - \varphi_{n_2+1})$. The other contributions from the vanishing of the various factors $\langle \tilde V_4, \tilde V_1\rangle$ (2 contributions depending on the position of the positive helicity gluon g_I), $\langle \tilde V_3, \tilde V_4\rangle$, $\langle \tilde V_2, \tilde V_3\rangle$. We will not shown the diagrams here. Their respective contributions are:

$$\tilde C_2 = \frac{(\alpha_1+\alpha_2)^3}{\alpha_1\alpha_2} \sum_{j=I}^{n_2-1} (\varphi_j - \varphi_{j+1})\,\langle \tilde v_0, \tilde V_2\rangle \,, \tag{A.19}$$

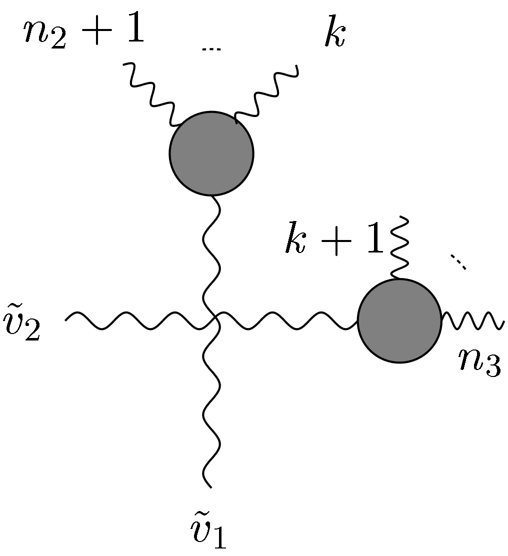

Figure 20: Contributing diagrams from the vanishing of $\langle \tilde{V}_1, \tilde{V}_2 \rangle$.

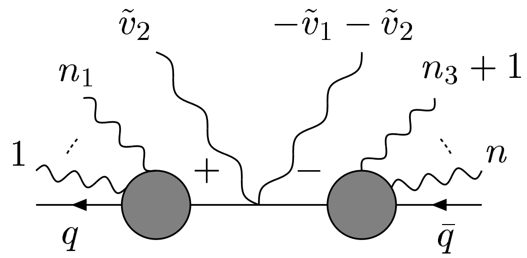

Figure 21: Contributing diagrams from the vanishing of $\langle V_2, V_3 \rangle$.

$$
\begin{aligned}
\tilde{C}_3 &= \frac{(\alpha_1+\alpha_2)^3}{\alpha_1\alpha_2} \sum_{j=n_1+1}^{I-1} (\varphi_j - \varphi_{j+1}) \langle \tilde{v}_0, \tilde{V}_2 \rangle \,, &\text{(A.20)}\\
\tilde{C}_4 &= \frac{(\alpha_1+\alpha_2)^3}{\alpha_1\alpha_2} \sum_{i=0}^{n_1-1} (\varphi_i - \varphi_{i+1}) \langle \tilde{v}_0, \tilde{V}_1 \rangle \,, &\text{(A.21)}\\
\tilde{C}_5 &= \frac{(\alpha_1+\alpha_2)^3}{\alpha_1\alpha_2} \sum_{l=n_3+1}^{n} (\varphi_l - \varphi_{l+1}) \langle \tilde{v}_0, \tilde{V}_4 \rangle \,, &\text{(A.22)}
\end{aligned}
$$

by omitting the same overall factor as in eq. (A.18). The last contribution is from the vanishing of the factor $\langle V_2, V_3 \rangle$ as shown in figure 21. It contribution is:

$$
\begin{aligned}
\tilde{C}_6 &= (\varphi_{\bar{q}} - \varphi_q) \frac{-\langle -\tilde{v}_1 - \tilde{v}_2, \tilde{V}_4 \rangle^2 \langle -\tilde{v}_1 - \tilde{v}_2, \tilde{v}_1 - \tilde{V}_4 \rangle}{\langle \tilde{v}_1 - \tilde{V}_4, \tilde{v}_2 \rangle \langle \tilde{V}_4, \tilde{v}_1 - \tilde{V}_4 \rangle}\\
&= \frac{(\alpha_1+\alpha_2)^3}{\alpha_1\alpha_2} (\varphi_{\bar{q}} - \varphi_q) \langle \tilde{v}_0, \tilde{V}_4 \rangle \,. &\text{(A.23)}
\end{aligned}
$$

By using all the above results, we have:

$$\sum_{i=1}^{6} \tilde{C}_i = 0 . \tag{A.24}$$

This shows that there is no finite pole terms.

Summarizing the above results from analyzing all the pole terms, we conclude that

$$F_f(\varphi) = a_1 \varphi_{n+1} + a_2 , \tag{A.25}$$

where a_1 and a_2 are functions of φ_0, φ_I and λ_s, $1 \leq s \leq n+1$, $s \neq I$. Then we can fixed a_1 and a_2 by a special set of φ_s, $1 \leq s \leq n+1$, $s \neq I$.

We can make a convenient choice such as: $\varphi_2 = \cdots = \varphi_{I-1} = x, \varphi_{I+1} = \cdots = \varphi_{n+1} = y$. After doing some algebras as in [12, 13], we have

$$F_f(\varphi) = (\varphi_q - \varphi_I)^3 (y - \varphi_I) . \tag{A.26}$$

Since y can take any value, we must have:

$$a_1 = (\varphi_q - \varphi_I)^3 , \qquad a_2 = -\varphi_I (\varphi_q - \varphi_I)^3 , \tag{A.27}$$

and so $F_f(\varphi) = (\varphi_q - \varphi_I)^3 (\varphi_{\bar{q}} - \varphi_I)$. This ends our proof of eq. (4.37).

References

[1] E. Witten, *Perturbative gauge theory as a string theory in twistor space*, hep-th/0312171.

[2] R. Penrose, *Twistor algebra*, *J. Math. Phys.* **8** (1967) 345; *The central programme of twistor theory*," *Chaos, Solitons, and Fractal* **10** (1999) 581.

[3] F. Cachazo, P. Svrcek and E. Witten, *MHV vertices and tree amplitudes in gauge theory*, *J. High Energy Phys.* **09** (2004) 006 [hep-th/0403047].

[4] S.J. Parke and T.R. Taylor, *An amplitude for N gluon scattering*, *Phys. Rev. Lett.* **56** (1986) 2459.

[5] F.A. Berends and W.T. Giele, *Recursive calculations for processes with N gluons*, *Nucl. Phys.* **B 306** (1988) 759.

[6] M.L. Mangano and S. Parke, *Multiparton amplitudes in guage theories*, *Phys. Rept.* **200** (1991) 30.

[7] Z. Bern, *String based perturbative methods for gauge theories*, hep-ph/9304249.

[8] L.J. Dixon, *Calculating scattering amplitudes efficiently*, hep-ph/9601359.

[9] Z. Bern, L.J. Dixon and D.A. Kosower, *Progress in one-loop QCD computations*, *Ann. Rev. Nucl. Part. Sci.* **46** (1996) 109–148 [hep-ph/9602280].

[10] C. Anastasiou, Z. Bern, L.J. Dixon and D.A. Kosower, *Planar amplitudes in maximally supersymmetric Yang-Mills theory*, *Phys. Rev. Lett.* **91** (2003) 251602 [hep-th/0309040]; Z. Bern, A. De Freitas and L.J. Dixon, *Two-loop helicity amplitudes for quark gluon scattering in QCD and gluino gluon scattering in supersymmetric yang- mills theory*, *J. High Energy Phys.* **06** (2003) 028 [hep-ph/0304168].

[11] V.P. Nair, *A current algebra for some gauge theory amplitudes*, *Phys. Lett.* **B 214** (1988) 215.

[12] C.-J. Zhu, *The googly amplitudes in gauge theory*, *J. High Energy Phys.* **04** (2004) 032 [hep-th/0403115].

[13] J.-B. Wu and C.-J. Zhu, *MHV vertices and scattering amplitudes in gauge theory*, *J. High Energy Phys.* **07** (2004) 032 [hep-th/0406085].

[14] R. Roiban, M. Spradlin and A. Volovich, *A googly amplitude from the B-model in twistor space*, *J. High Energy Phys.* **04** (2004) 012 [hep-th/0402016];
R. Roiban and A. Volovich, *All googly amplitudes from the B-model in twistor space*, hep-th/0402121.

[15] R. Roiban, M. Spradlin and A. Volovich, *On the tree-level S-matrix of Yang-Mills theory*, *Phys. Rev.* **D 70** (2004) 026009 [hep-th/0403190].

[16] N. Berkovits, *An alternative string theory in twistor space for $N = 4$ super-Yang-Mills*, *Phys. Rev. Lett.* **93** (2004) 011601 [hep-th/0402045].

[17] N. Berkovits and L. Motl, *Cubic twistorial string field theory*, *J. High Energy Phys.* **04** (2004) 056 [hep-th/0403187].

[18] M. Aganagic and C. Vafa, *Mirror symmetry and supermanifolds*, hep-th/0403192.

[19] E. Witten, *Parity invariance for strings in twistor space*, hep-th/0403199.

[20] A. Neitzke and C. Vafa, *$N = 2$ strings and the twistorial Calabi-Yau*, hep-th/0402128.

[21] N. Nekrasov, H. Ooguri and C. Vafa, *S-duality and topological strings*, hep-th/0403167.

[22] G. Georgiou and V.V. Khoze, *Tree amplitudes in gauge theory as scalar MHV diagrams*, *J. High Energy Phys.* **05** (2004) 070 [hep-th/0404072].

[23] S. Gukov, L. Motl and A. Neitzke, *Equivalence of twistor prescriptions for super Yang-Mills*, hep-th/0404085.

[24] W. Siegel, *Untwisting the twistor superstring*, hep-th/0404255.

[25] S. Giombi, R. Ricci, D. Robles-Llana and D. Trancanelli, *A note on twistor gravity amplitudes*, *J. High Energy Phys.* **07** (2004) 059 [hep-th/0405086].

[26] A.D. Popov and C. Saemann, *On supertwistors, the Penrose-ward transform and $N = 4$ super Yang-Mills theory*, hep-th/0405123.

[27] N. Berkovits and E. Witten, *Conformal supergravity in twistor-string theory*, *J. High Energy Phys.* **08** (2004) 009 [hep-th/0406051].

[28] I. Bena, Z. Bern and D.A. Kosower, *Twistor-space recursive formulation of gauge theory amplitudes*, hep-th/0406133.

[29] D.A. Kosower, *Next-to-maximal helicity violating amplitudes in gauge theory*, hep-th/0406175.

[30] F. Cachazo, P. Svrcek and E. Witten, *Twistor space structure of one-loop amplitudes in gauge theory*, hep-th/0406177.

[31] J.F. Gunion and Z. Kunszt, *Six quark subprocesses in QCD*, *Phys. Lett.* **B 176** (1986) 163.

[32] Xun Su and Jun-Bao Wu, *Six-Quark Amplitudes from Fermionic MHV Vertices*, to appear.

JHEP09(2004)063

[33] M.T. Grisaru, H.N. Pendleton and P. van Nieuwenhuizen, *Supergravity and the S matrix*, *Phys. Rev.* **D 15** (1977) 996.

[34] M.T. Grisaru and H.N. Pendleton, *Some properties of scattering amplitudes in supersymmetric theories*, *Nucl. Phys.* **B 214** (1977) 81.

[35] G. Georgiou, E.W.N. Glover and V.V. Khoze, *Non-MHV tree amplitudes in gauge theory*, *J. High Energy Phys.* **07** (2004) 048 [hep-th/0407027].

[36] M.L. Mangano, *The color structure of gluon emission*, *Nucl. Phys.* **B 309** (1988) 461.